MATH 402, MATH 500: Differential Geometry, Spring 2012, MWF 3-3:50 PM

Description: Differential geometry is the study of geometric figures using the methods of calculus. The principal object is a *smooth manifold*, which is a space similar enough to an ordinary Euclidean space to carry various structures and operations defined using differential calculus. One thinks of a smooth n dimensional manifold roughly as being covered by neighborhoods that are smoothly equivalent to regions in R^n. The somewhat technical precise definition involves some of the language of cartography. One speaks of a manifold having an *atlas* of *maps* or *coordinate charts* that give information on determining locations. Manifolds admit many extra structures. Especially important is the notion of a *Riemannian metric* which provides concepts of length, angle, volume, and many notions of curvature. As known to map makers, one cannot find an exactly distance preserving map of a region on the curved earth's surface onto a planar region. The differential geometric notions of curvature give the needed quantification of the distortions. There are numerous applications of differential geometry, both to other fields of mathematics such as algebraic geometry or topology and to other sciences and engineering, For example, in general relativity one considers 4 dimensional space-times that may not be globally R^4 or, when measuring distances precisely, even locally like a flat region in R^4 because gravitational souces contribute to the curvature of space-time. Some of the topics we may cover in the course include:

- Differential manifolds, Immersion and embedding,
- Vectorfields, brackets, integrability
- Differential forms, Poincaré Lemma, DeRham cohomology
- Riemannian metric, connections, differentiation
- Geodesics, convex neighborhoods, exponential map
- Harmonic forms, description of Hodge theory
- Curvature: Riemannian, sectional, Ricci, scalar
- Constant (sectional) curvature spaces
- Isometric immersion, second fundamental form
- Minimal surfaces, first variation, calibrating forms
- Manifolds of negative sectional curvature

Instructor: Robert Hardt, Herman Brown 430;
Office hours: Mon 1-2, Tues 11-12, Wed 1-2 (and others by appt.),
Email: hardt@rice.edu, Telephone: 3280
Text: M.P. doCarmo, *Riemannian Geometry*, Birkhauser, 1992.
Prerequisites: Some familiarity with multi-variable calculus, linear algebra, ordinary differential equations, and curves and surfaces.
Grading: Weekly homework 50%, Midterm 20%: Final 30%
Students registering for Math 500 will be requested to TeX some assignments.
Handouts: Fixed Point Theorem (http://math.rice.edu/ hardt/402-500S12/fpt.pdf) Inverse and Implicit Function Theorems (http://math.rice.edu/ hardt/402-500S12/ift.pdf)
Disabilities: Any student with a documented disability seeking academic adjustments or accommodations is requested to speak with me during the first two weeks of class. All such discussions will remain as confidential as possible. Students with disabilities will need to also contact Disability Support Services in the Allen Center.