Homework 10, due Tuesday, Dec.1

Problem 1. Show that each of the following formulas defines a *norm* on $\mathbf{R}^2$.

(a) $\| (x,y) \|_1 \ = \ |x| + |y|$.

(b) $\| (x,y) \|_2 \ = \ (|x|^2 + |y|^2)^{1/2}$.

(c) $\| (x,y) \|_3 \ = \ (|x|^3 + |y|^3)^{1/3}$.

(d) $\| (x,y) \|_\infty \ = \ \max\{|x|, \, |y|\}$.

Draw a picture of each of the four corresponding unit balls:

$$B^i \ = \ \{ \, (x,y) \ : \ \| \, (x,y) \, \|_i \ \leq \ 1 \, \} \ .$$

Problem 2. On the open first quadrant $U \ = \ \{(x,y) \ ; \ x > 0, \, y > 0\}$, consider P.D.E.

$$2x \, \frac{\partial u}{\partial y} \ - \ 2y \, \frac{\partial u}{\partial x} \ - \ 3u \ = \ 0$$

satisfying the condition $u(0,y) = y^5$ for all $y > 0$

(a) Find the characteristic curves for this problem.

(b) Can you solve this problem finding an explicit formula for the solution $u(x,y)$. If so, do it. If not, explain what other steps are needed.