Homework 2, due Thursday, Sept.11.

2.5, # 3, # 4(a)(b)(c)(d),

Extra Problem 1. Suppose u and h are twice continually differentiable functions on the closed ball $\bar{B}$ and

$$\int_B (\,|Du|^2 \;+\; hu)\,dx \leq \;\int_B (\,|Dv|^2 \;+\; hv)\,dx$$

for any twice continually differentiable function v on $\bar{B}$ with $v = u$ on ∂B. Find the PDE satisfied by u.

Extra Problem 2. **(Corrected)** Find a function $\Psi(x)$ on $\mathbf{R}^3$ so that for any smooth function f on $\mathbf{R}^3$, the function $u(x) = \int_{\mathbf{R}^3} \Psi(x-y)\, f(y)\, dy$ satisfies the PDE

$$\Delta\, u(x) = \;2f(x) \;+\; f\big(x + (1,0,0)\big)\;.$$

Some Hints:.

2.5#3 Try using the third Green's formula on page 628 with the region $\mathbf{B}(0,r)\setminus\mathbf{B}(0,\epsilon)$ and then let $\epsilon \to 0$.

2.5#4(b) Repeat the proof of 2.2.3 Theorem 4.