Homework 5, due Tuesday, Oct.7

2.5, # 12, Hint: Note that $\frac{d}{dt}(e^{At}u) = (e^{At}\frac{d}{dt}u) + A(e^{At})u$.

2.5, #14(a)(b)(c)(d),

Extra Problem 1. A polynomial of degree (at most) 3 in the variables x, y, t has the form

$$A + Bx + Cy + Dt + Ex^2 + Fy^2 + Gt^2 + Hxy + Ixt + Jyt$$

for some constants $A, B, \cdots, J$. Find all (at most) degree 3 polynomials, which are solutions of the heat equation $u_t - u_{xx} - u_{yy} = 0$.

Extra Problem 2. Read Example 1 on P.168 and consider the simplest case where $n = 1$, U is the open interval $(0, 2\pi)$, and $\Delta u = u_{xx} = \frac{d^2}{dx^2}u$. Then what are the eigenvalues λ_k and the eigenfunctions w_k of $-\Delta$? In equation (9) on page 169, how would you find the coefficients d_k from a given bounded smooth function g? Hint: Try to get w_k so that $\int_0^{2\pi} w_j w_k \, dx$ is zero for $j \neq k$ and 1 for $j = k$. (You need not discuss convergence. Just find the right formulas.)