Homework 6, due Thursday, Oct.16

2.5, # 15(a)(b)

2.5, #16

Extra Problem 1. Recall that a a real-valued function f on $\mathbf{R}$ belongs to C^k if f and its first k derivatives are all continuous functions. Show that:

(a) $|x|$ belongs to C^0 but not to C^1.

(b) $x|x|$ belongs to C^1 but not to C^2.

(c) $x^2|x|$ belongs to C^2 but not to C^3.

Hint: Find the formulas for the one-sided derivatives.

Extra Problem 2. Suppose that u is a smooth function on a bounded region U. Show that

u is harmonic if and only if $\int_U u(x)\Delta v(x)\, dx \ = \ 0$

for any smooth function v on U which vanishes near ∂U. Hint: Use integration by parts. For the "if" part, argue by contradiction and then choose a nonconstant, nonnegative v to have support in a region where Δu has constant nonzero sign.

Extra Problem 3. Suppose that a smooth functions u on $\mathbf{R}^n$ is a solution of a linear PDE

$$M[u] \equiv \sum_{|\alpha|\leq k} c_\alpha \, D^\alpha u \ = \ 0 \ ,$$

where the c_α are constants. Show that any partial derivative u_{x_i} is also a solution. That is, $M[u_{x_i}] = 0$.