Homework due Wed., Jan.31:

1. Find an example of a bounded continuous function u on the open interval $(-1, 1)$ such that $u \notin W^{1,2}$.

2. Suppose that U is a bounded smooth domain in $\mathbf{R}^n$ and $n_U : \partial U \to \mathbf{S}^{n-1}$ is the unit exterior-pointing normal. Show that for any $1 \leq p < \infty$, $u \in W^{1,p}(U)$, and $V \in \mathcal{C}^\infty(\mathbf{R}^n, \mathbf{R}^n)$

$$\int_{\partial U} (\mathbf{T}u)\big(V \cdot n_U\big) \;=\; \int_U u \operatorname{div} V \;+\; \int_U V \cdot Du \;.$$

Show that this formula determines the trace $\mathbf{T}u$ uniquely.

3. Suppose that for any $1 \leq p < \infty$, $u \in W^{1,p}(U)$, and $v \in W^{1,q}(U)$ with $\frac{1}{p} + \frac{1}{q} = 1$. Show that $uv \in W^{1,1}(U)$ and that the weak derivative $(uv)' = u'v + uv'$.