Homework due Wed., Feb.7:

1. P. 487, #1,

2. P. 487, #3

3. Find an example of a sequence $u_k \in W^{1,2}\big((0,1)\big)$ such that $u_k \rightharpoonup 0$ weakly in $W^{1,2}$ but not strongly in $W^{1,2}$. Hint: There is an example with $|u_k'| = 1$ almost everywhere.

4. Find an example of a sequence $v_k \in W^{1,1}\big((0,1)\big)$ with $\sup_k \|v_k\|_{W^{1,1}} < \infty$ such that *no* subsequence of v_k converges weakly in $W^{1,1}$. (This does not contradict the compactness theorem stated in class which referred to $W^{1,p}$ with $p > 1$.) Hint: Here it suffices to find a sequence $v_k \in W^{1,1}\big((0,1)\big)$ with $\sup_k \|v_k\|_{W^{1,1}} < \infty$ that converges in some larger space (e.g. $L^1(U)$) to a function v which does *not* belong to $W^{1,1}$.