Suppose $L(p,z,u) = |p|^3$ and U is the unit all in $\mathbf{R}^n$.
#1. Find the Euler-Lagrange equation satisfied by a minimizer of $\int_U |Du|^3\,dx$
#2. Show that $p \mapsto L(p,z,x)$ is convex for each $z \in \mathbf{R}$ and $x \in U$.
#3. Suppose that g is a smooth function on ∂U. Show that

$$\mathcal{A}_g \;=\; \{w \in W^{1,3}(U) \;:\; w = g \text{ on } \partial U\} \;\neq\; \emptyset\,.$$

That is, construct one function w such that $w = g$ on ∂U and $\int_U (|u|^3 + |Du|^3)\,dx \;<\; \infty$.
#4. Prove that $\mathcal{A}_g$ contains a minimizer $u \in W^{1,3}(U)$.
#5. Is this minimizer unique? Why or why not?