Homework due Wed., Feb.21:

Let w be a smooth positive function on $\mathbf{R}^2$ and

$$I[u] = \int_{-1}^{1} w\big(t, u(t)\big)\sqrt{1 + |u'(t)|^2}\, dt \ .$$

Suppose that u is a smooth function on $[-1, 1]$ and u minimizes I among smooth functions which vanish at the points $t = -1$ and $t = 1$.

1. Find the Euler-Lagrange equation satisfied by u.

2. Find u in case $w \equiv 1$.

3. Show that for some positive C and λ, the density function $w(t, z)) = C\, e^{-\lambda(t^2+z^2)}$ guarantees that there are at least 2 distinct such minimizers. Hint: you don't have to actually find these minimizers, just show that there is some nonzero v with $I[v] < I[0]$. You can make v using a semicircle.

4. P.488, #10 Hint: First find a $L(p, z, x)$ so that the given PDE is the Euler-Lagrange equation for $\int_U L(Du, u, x)dx$.