

Homework 7, due Wed. Feb.28

1. P.91, # 11

2. Suppose that U is a connected region in \mathbf{R}^n with smooth boundary and that f and u are smooth bounded functions on \bar{U} .

a) Show that u satisfies the two conditions:

$$-\Delta u = f \text{ on } U \quad \text{and} \quad \frac{\partial u}{\partial \nu} = 0 \text{ on } \partial U$$

if and only if

$$\int_U Du \cdot Dv \, dx = \int_U f v \, dx \text{ for all } v \in \mathcal{C}_c^\infty(\mathbf{R}^n). \quad (*)$$

b) Show that the existence of a solution u of (*) implies that $\int_U f \, dx = 0$.

c) Assuming $\int_U f \, dx = 0$, sketch a proof of how one would show the existence of a $W^{1,2}$ solution of (*). Hint: First find a functional to be minimized.