For #2(c) Observe that u is a solution of this "Neumann" problem if and only if $u+c$ is a solution for any constant c. so when forming a minimizing sequence it is good to normalize by adding the constraint to the admissible family that the Sobolev functions have integral 0 . Then one can use the Poincare inequality (see Evans p.275) as well as the Cauchy trick and Sobolev inequality to control the term $\int_U f u_k \, dx$.