Math 445, Spring 2010, Algebraic Topology

Meets: Monday-Wednesday-Friday, 2-2:50 PM, Hermann Brown Hall 423

Description: A *topological space* is simply a set X with a distinguished class $\mathcal{U}$ of subsets (called *open* which contains both $\emptyset$ and X and is closed under finite intersections and arbitrary unions. A bijection between topological spaces that preserves open sets both ways is called a *homeomorphism*. In **Algebraic Topology** one associates to a topological space many important algebraic objects (groups, rings, modules, etc.) each of which is invariant under homeomorphism. Thus one can verify that two spaces X and Y are topologically different, that is , non-homeomorphic if one can verify that $A(X) \neq A(Y)$ for any one such homeomorphic invariant A. One important example, well-studied in Math 444, is the *fundamental group* of X.

In Math 445, we study the next most popular invariants, the *homology* and *cohomology* of a topological space. These are typically sequences of abelian groups which roughly measure how groups of *cycles* (cocycles) of various dimensions are different from subgroups of *boundaries* (coboundaries). There are many interesting theories leading to different notions of cycle and boundary, which nevertheless lead to the same quotient homology groups. These are usually easier to compute than the possibly nonabelian *homotopy groups*, which are natural higher dimensional generalizations of the fundamental group. Though homology theory had its roots in the 19th century with work of Poincaré, the proper definitions and full axiomatization didn't occur until the 1950's with Eilenberg and Steenrod. See the article by Peter Hilton, "A Brief, Subjective History of Homology and Homotopy Theory in This Century", which you can access at http://www.jstor.org/stable/2689545 .

Some of the topics we will discuss include both simplicial and singular (co)homology theories, relative (co)homology, the Eilenberg-Steenrod axioms, the universal coefficient theorem, some basic homological algebra, Poincaré duality and the cohomology ring. If time permits, we may also treat the Kunneth formula and the DeRham and Cech cohomology theories.

Course Text: James Munkres, *Elements of Algebraic Topology*, Perseus Books Publishing, Cambridge, Massachusetts, 1984. ISBN: 0-201-62728.

Other useful textbooks:
    Allen Hatcher, *Algebraic Topology* Cambridge Univ. Press.
    M J Greenberg and J Harper, *Algebraic Topology: a First Course* (Benjamin/Cummings 1981)
    W S Massey, *A Basic Course in Algebraic Topology*, Springer Graduate Texts in Math 127
    A Dold, *Lectures on Algebraic Topology*, Springer-Verlag.

Homework (and also reading assignments) will be given and collected weekly and communicated via owlspace.rice.edu . First click on *Collaboration and Course Management* and then login with your NetID.

Grades are to be determined by the following: Homework, 40%, Midterm: 25% , Final: 35%

Prerequisites: Math 444 or equivalent

Instructor: Robert Hardt : Office: Herman Brown 430; Office hours: 11-12 MWF (and others by appt.), Email: hardt@rice.edu, Telephone: ext 3280

Disabilities: Any student with a documented disability seeking academic adjustments or accommodations is requested to speak with me during the first two weeks of class. All such discussions will remain as confidential as possible. Students with disabilities will need to also contact Disability Support Services in the Allen Center.