Descartes Circle Theorem

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The Descartes circle theorem is an ancient theorem about four mutually tangent circles in the Euclidean plane, asserting that their curvatures satisfy the quadratic relation,

\[ 2(\kappa_1^2 + \kappa_2^2 + \kappa_3^2 + \kappa_4^2) = (\kappa_1 + \kappa_2 + \kappa_3 + \kappa_4)^2. \]

Descartes, who lived in the period 1596-1650, discovered and proved this theorem, using Cartesian coordinates and algebra. The theorem was subsequently generalized to spheres in Euclidean space and other geometries (hyperbolic and spherical), and we’ll be able to present a complete proof of some of these results, basically using just algebra. (Thus we’ll be doing algebraic geometry!)