

Descartes Circle Theorem

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The Descartes circle theorem is an ancient theorem about four mutually tangent circles in the Euclidean plane, asserting that their curvatures satisfy the quadratic relation,

$$2(\kappa_1^2 + \kappa_2^2 + \kappa_3^2 + \kappa_4^2) = (\kappa_1 + \kappa_2 + \kappa_3 + \kappa_4)^2 .$$

Descartes, who lived in the period 1596-1650, discovered and proved this theorem, using *Cartesian* coordinates and algebra. The theorem was subsequently generalized to spheres in Euclidean space and other geometries (hyperbolic and spherical), and we'll be able to present a complete proof of some of these results, basically using just algebra. (Thus we'll be doing *algebraic geometry*!)