Please attempt each of the following problems. Use of computers, friends, and myself our encouraged.

1. Write out the full Taylor series for the curve
   \[ x^4 - y^5 + 5y^4 - 10y^3 + 10y^2 - 5y + 1 = 0 \]
   around the point (0, 1). What is the multiplicity of the singularity at this point?

Definition 0.1. Let \( X(f) \) be a curve with a multiplicity-two singularity. As hinted at during class, we call this singularity a cusp if the Hessian determinant of \( f \) is zero at the singularity and a node otherwise.

2. For each of the following plane curves, find all singular points and determine their multiplicities. If the multiplicity is two, determine if the point is a cusp or a node. Plot the curve near the singularity by using a program such as mathematica or maple with the command ImplicitPlot.

   \( x^2 y + xy^2 = x^4 + y^4 \)

Definition 0.2. A complex change of coordinates rescales our plane and changes the axes. A (complex) affine change of coordinates additionally shifts the origin, so is of the form \( \left( \begin{array}{c} x' \\ y' \end{array} \right) = \left( \begin{array}{cc} a & b \\ c & d \end{array} \right) \left( \begin{array}{c} x \\ y \end{array} \right) + \left( \begin{array}{c} e \\ f \end{array} \right) \).

4. Show that the curve \( x^2 - 2xy - x + y + 1/4 - y^3 = 0 \) is affine-equivalent to our good friend \( y^2 - x^2 - x^3 = 0 \).

5. Prove the following: \( y^2 = x^3 + px + q \) has no singular points if and only if \( x^3 + px + q \) has three distinct zeros. **Hint:** factor \( x^3 + px + q \).