Math 499 Exercises (03/01/06)

Here is what we really proved in class.

Theorem 0.1. *Let $F = 0$ be a smooth curve of degree $d \geq 3$. Let $\{p_1, \dots p_r\}$ be the set of flexes of F and let $\{L_1, \dots L_r\}$ be the corresponding tangent lines to F at our flexes. Then:*

$$\sum_{i=1}^{r} (I_{p_i}(F, L_i) - 2) \quad = \quad 3n(n-2).$$

1. For each of the folowing curves, find all the flexes (using the Hessian), and $I_{p_i}(F, L_i)$ at each point. Make sure the sum adds up correctly, according to the theorem. Don't compute the actual tangent lines (or even specific coordinates of points) if at all possible.

(a) $0 = x^4 + y^4 + z^4$
(b) $0 = x^3y + y^4 + z^4$
(c) Are these two curves projectively equivalent? Explain why or why not.

5. Do the same thing as above for the curve $z^4 = 2z^2y^2 + x^4$. What does this tell you?