The Interpolation Problem

The Fundamental Theorem of Algebra says that, given two sets of complex numbers $z_0, \ldots, z_d$ and $a_0, \ldots, a_d$, there exists a unique polynomial $f(z) \in \mathbb{C}[z]$ of degree d such that $f(z_i) = a_i$ for all i. We can extend this: given $z_1, \ldots, z_\delta$ and integers $m_1, \ldots, m_\delta$ with $\sum m_i = d + 1$, there exists a unique polynomial of degree d with arbitrary specified derivatives up to order m_i at z_i.

This is a beautiful, fundamental and highly useful result. But when we ask the natural next question—what can we say about polynomials in several variables—we enter a realm of mystery. The analogous statement for polynomials in two or more variables is visibly false, but no one knows exactly when, and by how much, it can fail. This gives rise to a whole class of problems, collectively known as *interpolation problems*.

The interpolation problem is like a number of problems in algebraic geometry: it's completely elementary to state; a general solution seems beyond us; and yet substantial progress has been made and is currently being made. In this talk I'll try to give an elementary introduction to the problem and what we know about it. In particular, I'll try to describe a common thread in the known and conjectured solutions of special cases, giving a geometric characterization of when interpolation fails in general.