On some two–dimensional configurations in dense sets

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A well–known result of Gowers asserts that any set $A \subseteq \{1, \dots, N\}$ of density at least $1/(\log\log N)^{c_k}$, $c_k > 0$ has an arithmetic progression of length k. It is easy to see that the last theorem implies that for an arbitrary finite set F there is an affine copy $aF + b$, where $a, b \in \mathbb{Z}$ which belongs to the set A. In our talk we consider several two–dimension generalizations of the last result for some specific sets F. Let us formulate our main theorem. Let $A \subseteq \{1, \dots, N\}^2$ be a subset of two–dimensional grid of the cardinality $|N|^2/(\log\log\log N)^c$, where $c > 0$ is an absolute constant. We prove that A contains a quadruple $\{(x, y), (x, y + r), (x + r, y), (x + 2r, y)\}$, for some $r \neq 0$. Thus the set F here is $\{(0, 0), (0, 1), (1, 0), (2, 0)\}$. Our result is a two–dimensional generalization of Gowers' theorem in the case $k = 4$. Also we obtain a similar statement for subsets A of the group $(\mathbb{Z}/p\mathbb{Z})^n \times (\mathbb{Z}/p\mathbb{Z})^n$, where n is a positive integer, and $p \geq 5$ is a prime number.

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