

del Pezzo Surface of degree 1 and 2 with few rational points

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One of the fundamental problems in Arithmetic Geometry is to determine the set of rational points $X(k)$ for a given algebraic variety X defined over a given field k . In other words, if the variety X could be described by some explicit equation, the question becomes : over any given field k , does there exist a solution to the equation?

Del Pezzo surfaces are smooth projective surfaces isomorphic over the algebraic closure of the base field k to $\mathbb{P}^1 \times \mathbb{P}^1$ or the blow-up of \mathbb{P}^2 in up to eight points in general position. For example, a cubic surface is del Pezzo surface of degree 3. We give an overview of the theory of del Pezzo surfaces, especially focus on the lower bound of the number of rational points.

For del Pezzo surfaces of degree 1, it is known that there is always one rational point over any given field k . Are there more points? How about del Pezzo surfaces of degree 2?

We will discuss the rational points on del Pezzo surface of degree 1 and 2 over any finite field \mathbb{F}_q , and give out the explicit equations of del Pezzo surfaces that have unique rational point.