Wilf Equivalence in the Generalized Factor Order

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Let $\mathbb{P}$ be the positive integers and consider the free monoid $\mathbb{P}^*$ of all words over $\mathbb{P}$. A word $w = w_1 w_2 \dots w_n$ has length $|w| = n$, norm $\Sigma(w) = \sum_i w_i$, and weight $wt(w) = (|w|, \Sigma(w))$. We say that w embeds u if there are words $w', v, w'' \in \mathbb{P}^*$ such that $w = w'vw''$, $|v| = |u|$, and $v_i \geq u_i$ for all $1 \leq i \leq |u|$. The generalized factor order is the order obtained by letting $u \leq w$ if and only if w embeds u. Let $\mathcal{E}(u)$ be the set of all words from $\mathbb{P}^*$ that embed u.

Two words u, v are Wilf equivalent ($u \sim_w v$) if there exists a weight preserving bijection between $\mathcal{E}(u)$ and $\mathcal{E}(v)$. We prove a nice sufficient condition for Wilf equivalence based on pebble diagrams, namely words with equivalent pebble diagrams are Wilf equivalent. Wilf equivalence necessitates a highly structured bijection between $\mathcal{E}(u)$ and $\mathcal{E}(v)$, and this structure is explored.