Elliptic Curves and the Congruent Number Problem

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In this presentation, I will use the congruent number problem to motivate a discussion of elliptic curves, focusing on the specific curve E_n defined by $y^2 = x^3 - n^2x$ in $\mathbb{P}^2_{\mathbb{C}}$ when needed. I will define the Weierstrass $\wp$-function, demonstrate how it leads to a group addition law on an elliptic curve, adn explain the geometry of this addition law. Finally, I will discuss the algebra of points of finite order on E_n and show that it leads to the following characterization of congruent numbers:

$$n \in \mathbb{Z}^+ \text{ is congruent } \Leftrightarrow \text{ rank}(E_n) > 0.$$