

# MATH 499 Exercises

September 12, 2007

1. For each of the following curves, calculate the partial derivatives and find the singular points. Then calculate the Taylor expansion for the curve about each singular point. If possible, identify each singularity as a cusp or a node. Finally plot each curve (using Maple, for example, with the command “implicitplot”).
  - (i)  $x^2 = x^4 + y^4$
  - (ii)  $xy = x^6 + y^6$
  - (iii)  $x^3 = y^2 + x^4 + y^4$
  - (iv)  $x^2y + xy^2 = x^4 + y^4$
  - (v)  $y^2 = x^3 + x^2$
  - (vi)  $(x^2 + y^2)^2 = x^2 - y^2$
2. Prove that  $y^2 = x^3 + px + q$  has no singular points if and only if  $f(x) = x^3 + px + q$  has three distinct roots. [Hint: First show that a polynomial  $f(x)$  has a multiple root at  $a$  if and only if  $f(a) = f'(a) = 0$ .]