MATH 499 Exercises

February 13, 2008

1. For this problem, let R be a ring.

   (i) Show that $1-x$ is invertible in $R[[x]]$, with inverse $1+x+x^2+\cdots$.

   (ii) Show that $\sum_{n=0}^{\infty} a_n x^n$, $a_i \in R$, is invertible in $R[[x]]$ if and only if a_0 is invertible.

2. Show that $f(x,y) = x^2 - y^3$ and $g(u,v) = u^2 + u^4 - v^3$ are equivalent, but neither are equivalent to $h(s,t) = st$.

3. Let $f(x) = c_m x^m + c_{m+1}x^{m+1} + \cdots \in \mathbb{C}[[x]]$ with $c_m \neq 0$. Show that f is analytically equivalent to $g(z) = z^m$ at the origin.

   [Hint: Re-write f as $f = x^m + c_{m+1}x^{m+1} + \cdots$ using some analytic function. Consider a partial sum $x^m + c_{m+1}x^{m+1}$ and make the substitution $x = z + \alpha z^2$. Choose α to cancel the $(m+1)$-degree term. Then show that this can be repeated to cancel higher-degree terms.]

4. Show that $f(x_1,\ldots,x_n) = x_1^2 + \cdots + x_n^2 + f_3(x_1,\ldots,x_n) + \cdots \in \mathbb{C}[[x_1,\ldots,x_n]]$ (that is, a formal power series whose second order homogeneous term is $f_2 = x_1^2 + \cdots + x_n^2$) is analytically equivalent at the origin to $g(z_1,\ldots,z_n) = z_1^2 + \cdots + z_n^2$.