Math 211 - Hw #5

B 3/4.1 By direct substitution, one checks that

$$e^t \cos t, \quad e^t \sin t, \quad c_1 e^t \cos t + c_2 e^t \sin t$$

satisfy $y'' - 2y' + 2y = 0$

B 11/4.1 By direct substitution, one can check that

$$y_1(t) = \cos 4t \quad \text{and} \quad y_2(t) = \sin 4t$$

are solutions of $y'' + 16y = 0$. Moreover, these functions are linearly independent (neither is a constant multiple of the other); one can also compute the Wronskian for this:

$$W(\cos 4t, \sin 4t) = \begin{vmatrix} \cos 4t & \sin 4t \\ -4\sin 4t & 4\cos 4t \end{vmatrix} =$$

$$= 4\cos^2 4t + 4\sin^2 4t = 4 \neq 0.$$

The general solution is given by

$$y(t) = c_1 \cos 4t + c_2 \sin 4t$$

We find c_1, c_2 from $y(0) = 2$, $y'(0) = -1$:

$$y(0) = 2 \Rightarrow c_1 = 2$$

$$y'(0) = -1 \Rightarrow 4c_2 = -1 \Rightarrow c_2 = -1/4$$

Hence $y(t) = 2\cos 4t - \frac{1}{4}\sin 4t.$

B 9/4.3. $y'' + 4y' + 5y = 0$

Characteristic equation: $\lambda^2 + 4\lambda + 5 = 0$

$$\lambda = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm 2i}{2} \begin{cases} \lambda_1 = -2+i \\ \lambda_2 = -2-i \end{cases}$$

Fundamental set of solutions: $e^{-2t}\cos t$, $e^{-2t}\sin t$

General solution: $c_1 \cdot e^{-2t}\cos t + c_2 \cdot e^{-2t}\sin t$

B 13/4.3 $y'' - 4y' + 4y = 0$

$\lambda^2 - 4\lambda + 4 = 0$ has repeated root $\lambda = 2$

Fundamental set of solutions: e^{2t}, $t \cdot e^{2t}$

General solution: $c_1 e^{2t} + c_2 t \cdot e^{2t}$

B 22/4.3. $y'' + 10y' + 25y = 0$ $y(0) = 2$, $y'(0) = -1$

$\lambda^2 + 10\lambda + 25 = 0$ has repeated root $\lambda = -5$

General solution $y(t) = c_1 \cdot e^{-5t} + c_2 \cdot t e^{-5t}$

From $y(0) = 2 \Rightarrow c_1 = 2$

$y'(0) = -1 \Rightarrow -5c_1 + c_2 = -1 \Rightarrow c_2 = 9$

Hence $y(t) = 2 \cdot e^{-5t} + 9 \cdot t \cdot e^{-5t}$

B13/4.1 Similarly to the previous problem, by direct substitution, one shows that $y_1(t) = e^{-4t}$, $y_2(t) = t \cdot e^{-4t}$ are solutions of $y'' + 8y' + 16y = 0$. These solutions are linearly independent, since the Wronskian is nonzero:

$$W(y_1(t), y_2(t)) = \begin{vmatrix} e^{-4t} & t \cdot e^{-4t} \\ -4e^{-4t} & e^{-4t} - 4t \cdot e^{-4t} \end{vmatrix} =$$

$$= e^{-8t} - 4t \cdot e^{-8t} + 4t \cdot e^{-8t}$$

$$= e^{-8t} \neq 0$$

Hence, the general solution is $y(t) = c_1 \cdot e^{-4t} + c_2 \, t \cdot e^{-4t}$

Since $y(0) = 2$, $y'(0) = -1$, one can find $c_1 = 2$, $c_2 = 7$,

and $y(t) = 2 \cdot e^{-4t} + 7t \cdot e^{-4t}$.

B 4/4.3. $y'' + 5y' + 6y = 0$

Characteristic equation: $\lambda^2 + 5\lambda + 6 = 0$

$$\lambda = \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm 1}{2} \begin{cases} \lambda_1 = -3 \\ \lambda_2 = -2 \end{cases}$$

Fundamental set of solutions: e^{-3t}, e^{-2t}

General solution: $c_1 \cdot e^{-3t} + c_2 \cdot e^{-2t}$.

B 28/4.3 $y'' - 4y' + 13y = 0 \quad y(0) = 4, \; y'(0) = 0$

$\lambda^2 - 4\lambda + 13 = 0$ has complex roots $\lambda = 2 \pm 3i$.

The general solution : $y(t) = c_1 \cdot e^{2t} \cos 3t + c_2 \cdot e^{2t} \sin 3t$

From $y(0) = 4 \Rightarrow c_1 = 4$

From $y'(0) = 0 \Rightarrow 2c_1 + 3c_2 = 0 \Rightarrow c_2 = -8/3$

Therefore $y(t) = 4 \cdot e^{2t} \cos 3t - \frac{8}{3} e^{2t} \sin 3t.$