Hw # 8 - Math 211

B 11/7.1. $a=-3$, $A=\begin{bmatrix}-1 & 2 & 0\\ 3 & 0 & -1\\ 2 & 2 & 1\end{bmatrix}$, $\bar{x}=\begin{bmatrix}1\\-2\\0\end{bmatrix}$; $\bar{y}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$

Need to verify: 1) $A(a\bar{x})=aA\bar{x}$

2) $A(\bar{x}+\bar{y})=A\bar{x}+A\bar{y}$

1) $$A(a\bar{x})=\begin{bmatrix}-1 & 2 & 0\\ 3 & 0 & -1\\ 2 & 2 & 1\end{bmatrix}\left((-3)\cdot\begin{bmatrix}1\\-2\\0\end{bmatrix}\right)=\begin{bmatrix}-1 & 2 & 0\\ 3 & 0 & -1\\ 2 & 2 & 1\end{bmatrix}\begin{bmatrix}-3\\6\\0\end{bmatrix}=\begin{bmatrix}15\\-9\\6\end{bmatrix}$$

$$aA\bar{x}=(-3)\begin{bmatrix}-1 & 2 & 0\\ 3 & 0 & -1\\ 2 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\-2\\0\end{bmatrix}=(-3)\begin{bmatrix}-5\\3\\-2\end{bmatrix}=\begin{bmatrix}15\\-9\\6\end{bmatrix}$$

2) $$A(\bar{x}+\bar{y})=\begin{bmatrix}-1 & 2 & 0\\ 3 & 0 & -1\\ 2 & 2 & 1\end{bmatrix}\left(\begin{bmatrix}1\\-2\\0\end{bmatrix}+\begin{bmatrix}0\\-1\\4\end{bmatrix}\right)=\begin{bmatrix}-1 & 2 & 0\\ 3 & 0 & -1\\ 2 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\-3\\4\end{bmatrix}=\begin{bmatrix}-7\\-1\\0\end{bmatrix}$$

$$A\bar{x}+A\bar{y}=\begin{bmatrix}-1 & 2 & 0\\ 3 & 0 & -1\\ 2 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\-2\\0\end{bmatrix}+\begin{bmatrix}-1 & 2 & 0\\ 3 & 0 & -1\\ 2 & 2 & 1\end{bmatrix}\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}-5\\3\\-2\end{bmatrix}+\begin{bmatrix}-2\\-4\\-2\end{bmatrix}=\begin{bmatrix}-7\\-1\\0\end{bmatrix}$$

B35/7.1. $$\begin{bmatrix}-6 & -1 & 7\\ -1 & 8 & -9\end{bmatrix}\begin{bmatrix}-1\\0\\2\end{bmatrix}=\begin{bmatrix}20\\-17\end{bmatrix}$$

B46/7.1. $$D^T=\begin{bmatrix}10 & -5 & 3\\ 0 & 8 & 6\\ 0 & -9 & 7\\ -1 & 3 & 6\end{bmatrix}$$

B 53/7.1 $$\begin{bmatrix}-1 & 0 & 1\\ 2 & 3 & 0\\ 0 & 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\4\end{bmatrix}$$

B 3/7.2. The set $S=\left\{\begin{bmatrix}t\\0\end{bmatrix} \mid t>0\right\}$ cannot be a solution of a linear system, because S is only a half-line, not a full line or a point.

B 7/7.2. $\bar{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Since this represents a line in $\mathbb{R}^2$, one would expect to find a linear system having $\bar{y}$ as solution set. Indeed, if $\bar{y} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, then

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \text{ hence}$$

$$x_1 = 2t$$
$$x_2 = 1 - 3t$$

We have $t = x_1/2$ and $x_2 = 1 - 3\frac{x_1}{2}$, or

$$\frac{3x_1}{2} + x_2 = 1, \text{ or } 3x_1 + 2x_2 = 2$$

The linear equation $3x_1 + 2x_2 = 2$ has $\bar{y}$ as the general solution.

B 5/7.3 Apply the row reduction procedure:

$$\left[\begin{array}{cc|c} \boxed{2} & -3 & 0 \\ 1 & -4 & 2 \end{array}\right] \underset{-\frac{1}{2}R_1 \text{ to } R_2}{\overset{\text{add}}{\sim}} \left[\begin{array}{cc|c} 2 & -3 & 0 \\ 0 & -\frac{5}{2} & 2 \end{array}\right] \leftarrow \text{row echelon form}$$

$$\begin{aligned} 2x - 3y &= 0 \\ -\tfrac{5}{2}y &= 2 \end{aligned} \xrightarrow{\text{Backsolving}} y = -\frac{4}{5}, \quad x = \frac{3y}{2} = -\frac{6}{5}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6/5 \\ -4/5 \end{bmatrix}$$

B 13/7.3

$$\left[\begin{array}{ccc|c} 4 & 7 & 5 & 18 \\ -2 & 1 & -1 & 0 \end{array}\right] \underset{\frac{1}{2}R_1 \text{ to } R_2}{\overset{\text{add}}{\sim}} \left[\begin{array}{ccc|c} \boxed{4} & 7 & 5 & 18 \\ 0 & \boxed{9/2} & 3/2 & 9 \end{array}\right] \leftarrow \text{row echelon form}$$

$$\begin{aligned} 4x + 7y + 5z &= 18 \\ \tfrac{9}{2}y + \tfrac{3}{2}z &= 9 \end{aligned} \quad \xrightarrow{\text{backsolving}} \quad y = \frac{9 - \frac{3}{2}t}{\frac{9}{2}} = 2 - \frac{1}{3}t$$

z - free variable $(= t)$

$$x = \frac{18 - 5z - 7y}{4} = \frac{18 - 5t - 7\cdot\left(2 - \frac{1}{3}t\right)}{4}$$

$$= \frac{4 - \frac{8}{3}t}{4} = 1 - \frac{2}{3}t$$

Hence $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 - \frac{2}{3}t \\ 2 - \frac{1}{3}t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2/3 \\ -1/3 \\ 1 \end{bmatrix}$

B 23/7.3

$$\left[\begin{array}{ccc|c} -12 & 12 & -8 & -8 \\ -16 & 16 & -10 & -10 \\ -3 & 3 & -1 & -1 \end{array}\right] \underset{\sim}{R_1 \leftrightarrow R3} \left[\begin{array}{ccc|c} -3 & 3 & -1 & -1 \\ -16 & 16 & -10 & -10 \\ -12 & 12 & -8 & -8 \end{array}\right]$$

add $-\frac{16}{3}R_1$ to R_2
$\sim$
add $-\frac{12}{3}R_1$ to R_3

$$\left[\begin{array}{ccc|c} \boxed{-3} & 3 & -1 & -1 \\ 0 & 0 & \boxed{-14/3} & -14/3 \\ 0 & 0 & -4 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{-3} & 3 & -1 & -1 \\ 0 & 0 & \boxed{-14/3} & -14/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$-3x_1 + 3x_2 - x_3 = -1$$
$$-14/3\, x_3 = -14/3$$

x_2 - free variable
$x_2 = t$

$x_3 = 1$ and $x_1 = \dfrac{-1 + x_3 - 3x_2}{-3} = \dfrac{-1 + 1 - 3t}{-3}$
$= t$

Hence $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

B 24/7.3 $\left[\begin{array}{cccc|c} \boxed{-4} & 10 & 4 & -8 & 6 \\ 0 & \boxed{4} & 6 & -7 & 4 \end{array}\right]$ Already in row echelon form

$$-4x_1 + 10x_2 + 4x_3 - 8x_4 = 6$$
$$4x_2 + 6x_3 - 7x_4 = 4$$

$\longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -11/4 \\ -3/2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 19/8 \\ 7/4 \\ 0 \\ 1 \end{bmatrix}$

x_3, x_4 - free variables $(x_3 = s, x_4 = t)$