17/1.3. \( x' = t \cdot e^{-t^2} \), \( x(0) = 1 \)

Integrate: \( x(t) = \int t \cdot e^{-t^2} \, dt = -\frac{1}{2} \cdot e^{-t^2} + C \)

From \( x(0) = 1 \), we get \( C = \frac{3}{2} \).

Hence \( x(t) = -\frac{1}{2} \cdot e^{-t^2} + \frac{3}{2} \).

28/1.3. Need to solve \( y''(t) = -g \) with \( y(0) = 1000 \), \( y'(0) = -25 \).

Integrate twice the equation: \( y(t) = -\frac{1}{2} gt^2 + C_1 t + C_2 \)

Use initial conditions to get: \( C_1 = 25 \), \( C_2 = 1000 \).

Determine \( t \) such that \( y(t) = 0 \) (when it hits the ground): \(-\frac{1}{2} gt^2 - 25t + 1000 = 0 \) (\( g = 9.8 \))

This gives \( t = 11.96 \) seconds.
B 3/2.1 By direct computation, one shows that
\[ y'(t) = Ce^{-(1/2)t^2} \frac{1}{2} (-2t) = -ty(t). \]
The family of solutions for \( C = -3, -2, \ldots, 3 \) is plotted below.

B 11/2.1 By direct computation, one shows that \( y(t) = \frac{4}{1 + 5e^{-4t}} \) satisfies \( y' = y(4-y) \) and \( y(0) = -1 \). For the function \( y(t) \) to be well defined, \( t \neq (\ln 5)/4 \). Since the initial condition is \( y(0) = 1 \), the interval of existence is \((-\infty, (\ln 5)/4)\).
At each integer valued coordinates \((t, y)\) draw a short line of slope \(f(t, y) = y^2 - t\). For example at \((-2, -1)\) the slope is 3, etc.

\[y' = 1 - y + 1\]

34/2.1. Same as 3/2.1.
\[ \frac{dy}{dx} = (1 + y^2) e^x \]

Separate variables: \( \frac{dy}{1 + y^2} = e^x \, dx \)

Integrate: \[ \int \frac{dy}{1 + y^2} = \int e^x \, dx \]

\[ \arctan y = e^x + C \]

Thus \[ y(x) = \tan(e^x + C) \]

\[ y^3 \frac{dy}{dx} = x + 2y \]

Rewrite as \[ y^3 \frac{dy}{dx} - 2y \frac{dy}{dx} = x \text{ or } y \frac{dy}{dx} (y^3 - 2) = x \text{ or } \]

\[ \frac{dy}{dx} (y^3 - 2) = x \]

Separate variables: \( (y^3 - 2) \, dy = x \, dx \)

Integrate: \[ \int (y^3 - 2) \, dy = \int x \, dx \]

\[ \frac{y^4}{4} - 2y = \frac{1}{2} x^2 + C \text{ or } \]

\[ \frac{y^4}{4} - 2y - \frac{1}{2} x^2 = C \]

Implicit solution

(not possible to find \( y(x) \) explicitly)
\[
\frac{dy}{dt} = -2t(y + 1) / y \quad y(0) = 1
\]

\[
\int \frac{y}{1+y^2} \, dy = \int -2t \, dt
\]

\[
\frac{1}{2} \ln (1+y^2) = -t^2 + C
\]

\[
1 + y^2 = e^{-2t^2 + 2C}
\]

Use \( y(0) = 1 \) to get \( e = 2, \ C = \ln 2 / 2 \)

\[
1 + y^2 = e^{-2t^2 + \ln 2} = e^{-2t^2}
\]

\[
y''(t) = 2 \cdot e^{-2t^2} - 1
\]

\[
y(t) = \pm \sqrt{2 \cdot e^{-2t^2} - 1}
\]

Because \( y(0) = 1 \) we need \( y(t) = \sqrt{2 \cdot e^{-2t^2} - 1} \)

Interval of existence: need \( 2 \cdot e^{-2t^2} - 1 > 0 \) so

\[
2 \cdot e^{-2t^2} > 1 \Rightarrow e^{-2t^2} > \frac{1}{2} \text{ or } -2t^2 > \ln \frac{1}{2}
\]

\[
\Rightarrow t^2 < -\frac{\ln \frac{1}{2}}{2} = \frac{\ln 2}{2} \Rightarrow \left(-\frac{\ln 2}{2}, \frac{\ln 2}{2}\right)
\]
>> dsolve('Dy=1+y^2', 'y(0)=1')

ans = tan(t+1/4*pi)

>> ezplot(ans,[-pi,pi])
y = dsolve('Dy = y*sin(t)', 'y(0) = 1');
ezplot(y, [-2*pi, 2*pi])
xlabel('t')
ylabel('y(t)')
title('Solution to y' = y*sin(t), y(0) = 1')