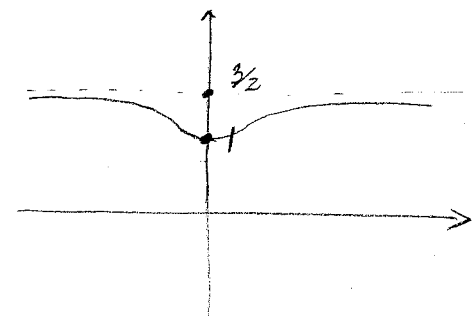
Homework #1

17/1.3. $x' = t \cdot e^{-t^2}$, $x(0) = 1$

Integrate: $x(t) = \int t \cdot e^{-t^2}\,dt = -\frac{1}{2} \cdot e^{-t^2} + c$

From $x(0) = 1$, we get $c = \frac{3}{2}$.

Hence $\underline{x(t) = -\frac{1}{2} \cdot e^{-t^2} + \frac{3}{2}}$



28/1.3 Need to solve $y''(t) = -g$ with $y(0) = 1000$, $y'(0) = -25$

Integrate twice the equation: $y(t) = -\frac{1}{2} g t^2 + c_1 t + c_2$

Use initial conditions to get: $c_1 = 25$; $c_2 = 1{,}000$.

Determine t such that $y(t) = 0$ (when it hits the ground)

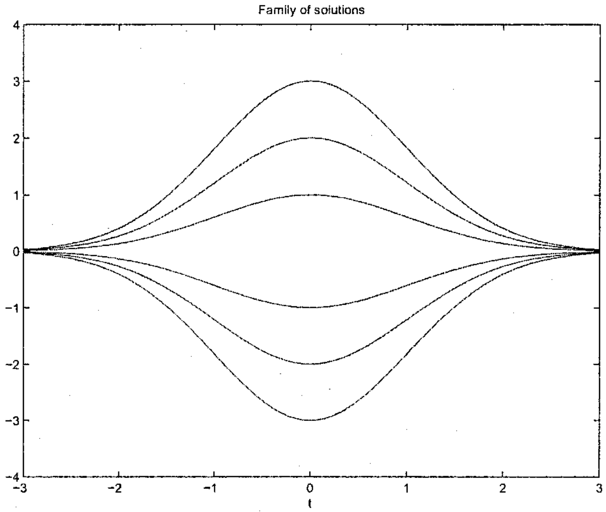
$-\frac{1}{2} g t^2 - 25t + 1000 = 0$ $(g = 9.8)$

gives $\underline{t = 11.96 \text{ seconds}}$

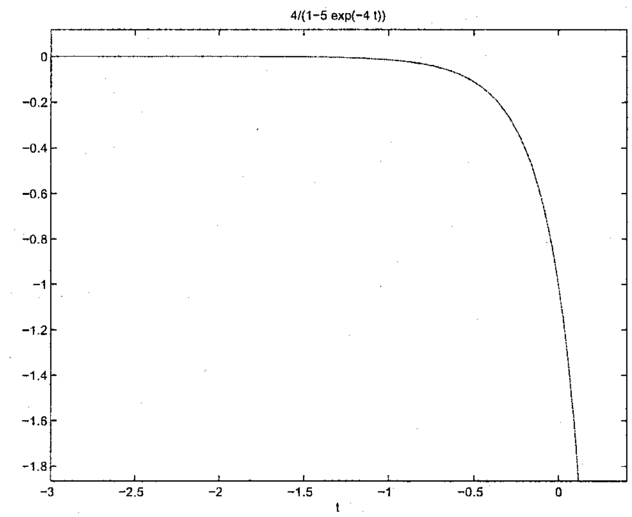
B 3/2.1 By direct computation, one shows that

$$y'(t) = Ce^{-(1/2)t^2}\frac{1}{2}(-2t) = -ty(t)\,.$$

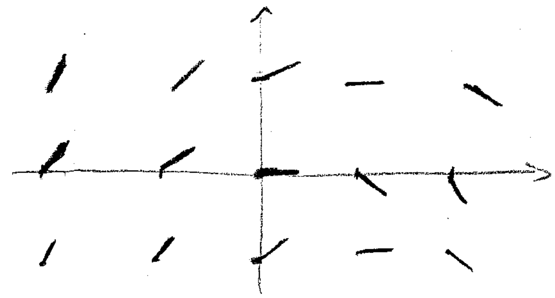
The family of solutions for $C = -3, -2, \ldots, 3$ is plotted below.



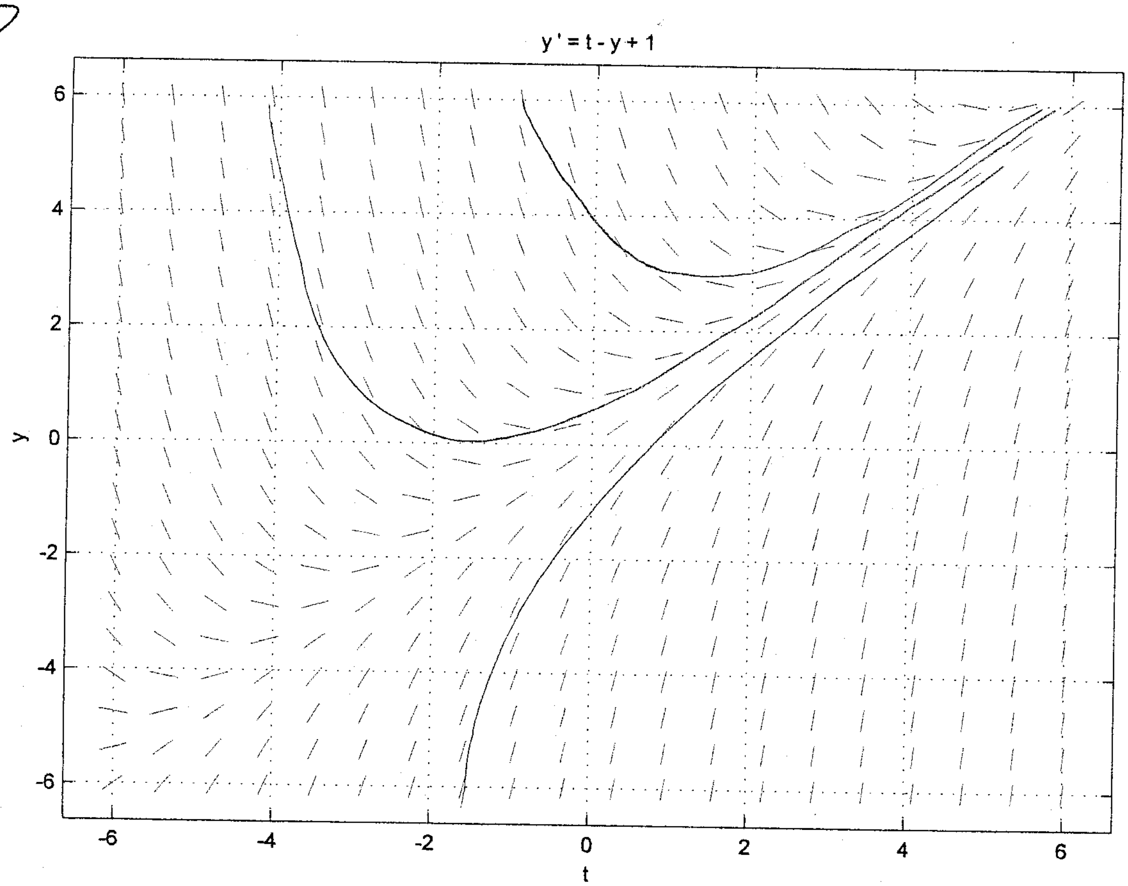
B 11/2.1 By direct computation, one shows that $y(t) = 4/(1 - 5e^{-4t})$ satisfies $y' = y(4 - y)$ and $y(0) = -1$. For the function $y(t)$ to be well defined, $t \neq (\ln 5)/4$. Since the initial condition is $y(0) = 1$, the interval of existence is $(-\infty, (\ln 5)/4)$.



18/2.1 At each integer valued coordinates (t, y) draw a short line of slope $f(t,y) = y^2 - t$. For example at $(-2,-1)$ the slope is 3, etc.



23/2.1



34/2.1. Same as 3/2.1.

4/2.2 $\quad \frac{dy}{dx} = (1+y^2)e^x$

Separate variables: $\frac{dy}{1+y^2} = e^x\,dx$

Integrate: $\int \frac{dy}{1+y^2} = \int e^x\,dx$

$$\arctan y = e^x + C$$

Thus $\boxed{y(x) = \tan(e^x + C)}$

11/2.2 $\quad y^3 y' = x + 2y'$

Rewrite as $y^3 y' - 2y' = x$ or

$$y'(y^3 - 2) = x \quad \text{or}$$

$$\frac{dy}{dx}(y^3 - 2) = x$$

Separate variables: $(y^3 - 2)\,dy = x\,dx$

Integrate: $\int (y^3 - 2)\,dy = \int x\,dx$

$$\frac{y^4}{4} - 2y = \frac{1}{2}x^2 + C \quad \text{or}$$

$$\underline{\frac{y^4}{4} - 2y - \frac{1}{2}x^2 = C}$$

implicit solution
(not possible to find $y(x)$ explicitly)

14/2.2 $\quad \frac{dy}{dt} = -2t(1+y^2)/y \quad y(0)=1$

$$\int \frac{y}{1+y^2}\,dy = \int -2t\,dt$$

$$\frac{1}{2}\ln(|1+y^2|) = -t^2 + C$$

$$1+y^2 = e^{-2t^2+2C}$$

Use $y(0)=1$ to get $e^{2C}=2$, $C=\ln 2/2$

So $1+y^2 = e^{-2t^2+\ln 2} = 2\cdot e^{-2t^2}$

$$y^2(t) = 2\cdot e^{-2t^2} - 1$$

$$y(t) = \pm\sqrt{2\cdot e^{-2t^2} - 1}$$

Because $y(0)=1$ we need $\boxed{y(t) = \sqrt{2\cdot e^{-2t^2} - 1}}$

Interval of existence: need $2\cdot e^{-2t^2} - 1 > 0$ so

$2\cdot e^{-2t^2} > 1 \Rightarrow e^{-2t^2} > 1/2$ or $-2t^2 > \ln\frac{1}{2}$

$$\Rightarrow \quad t^2 < -\frac{\ln\frac{1}{2}}{2} = \frac{\ln 2}{2} \Rightarrow \boxed{t \in \left(-\sqrt{\frac{\ln 2}{2}}, \sqrt{\frac{\ln 2}{2}}\right)}$$

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>> dsolve('Dy=1+y^2', 'y(0)=1')

ans = tan(t+1/4*pi)

>> ezplot(ans,[-pi,pi])
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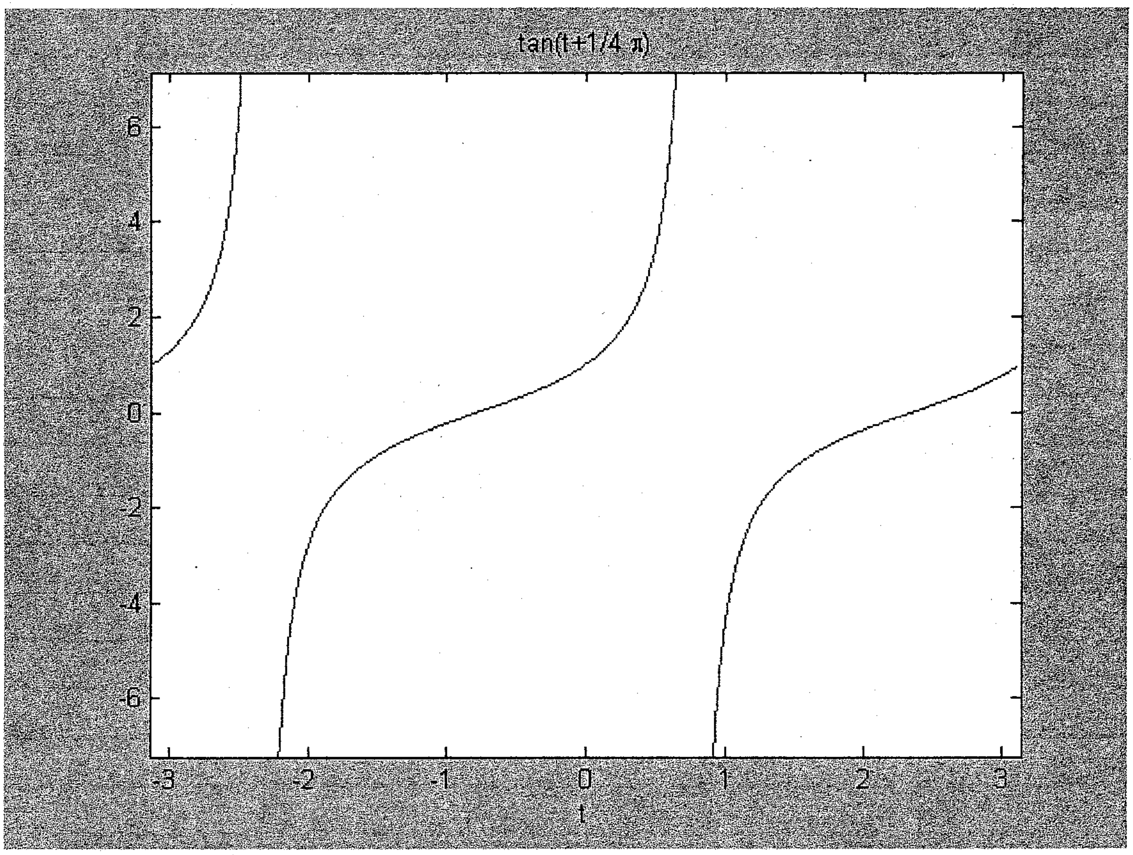

M 21/ch. 2


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y=dsolve('Dy=y*sin(t)','y(0)=1');
ezplot(y,[-2*pi,2*pi])
xlabel('t')
ylabel('y(t)')
title('Solution to y''=ysin(t), y(0)=1')
```