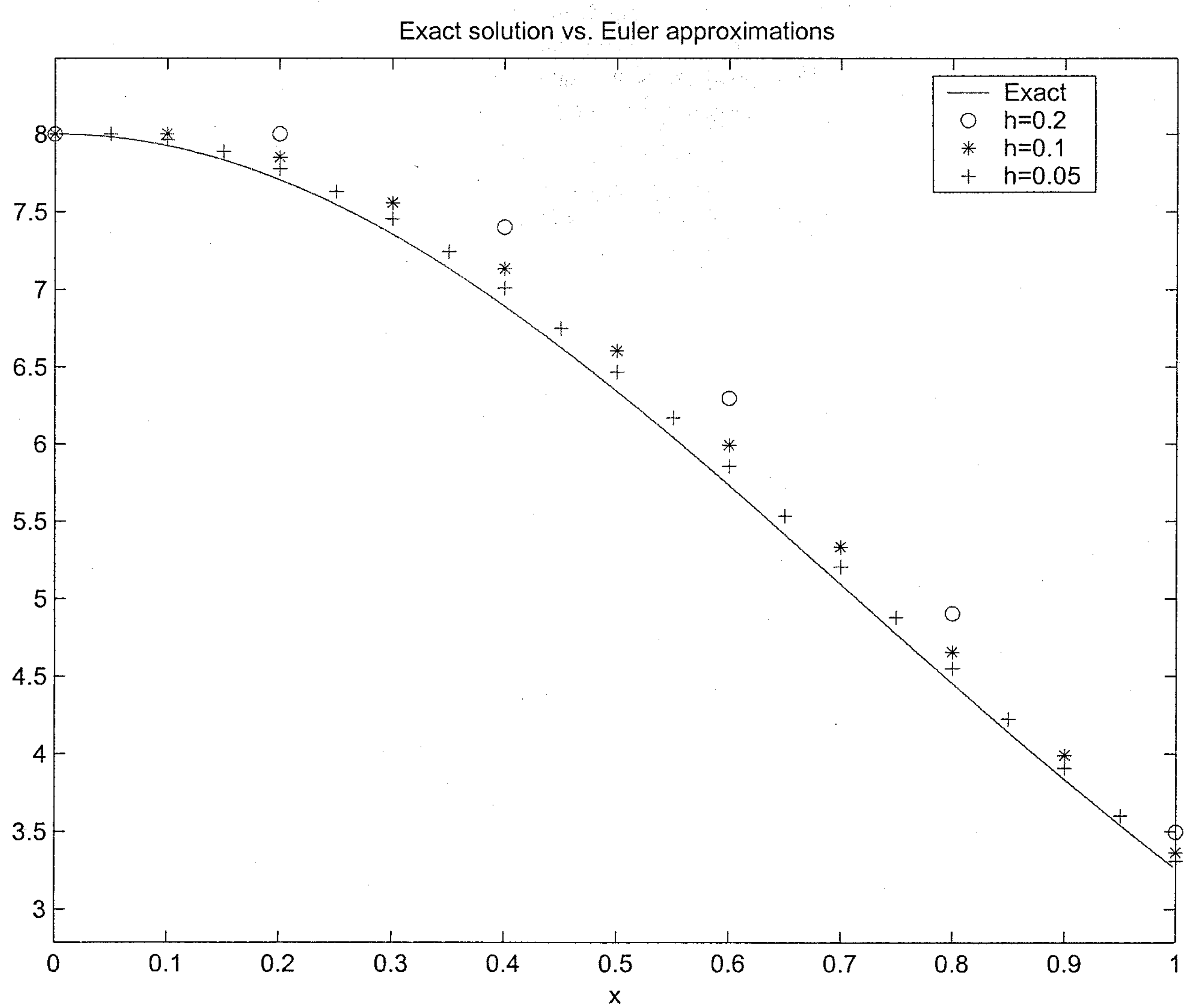
Math 211 - Hw #5

5/6.1 See table at the end of the book.

$\boxed{7/6.1}$ (ii) Solve $y' = -2xy + x$ $y(0) = 8$ as a linear equation (or separable) to get

$$y(x) = \frac{1}{2} + \frac{15}{2} \cdot e^{-x^2}$$

(i), (iii) See graph



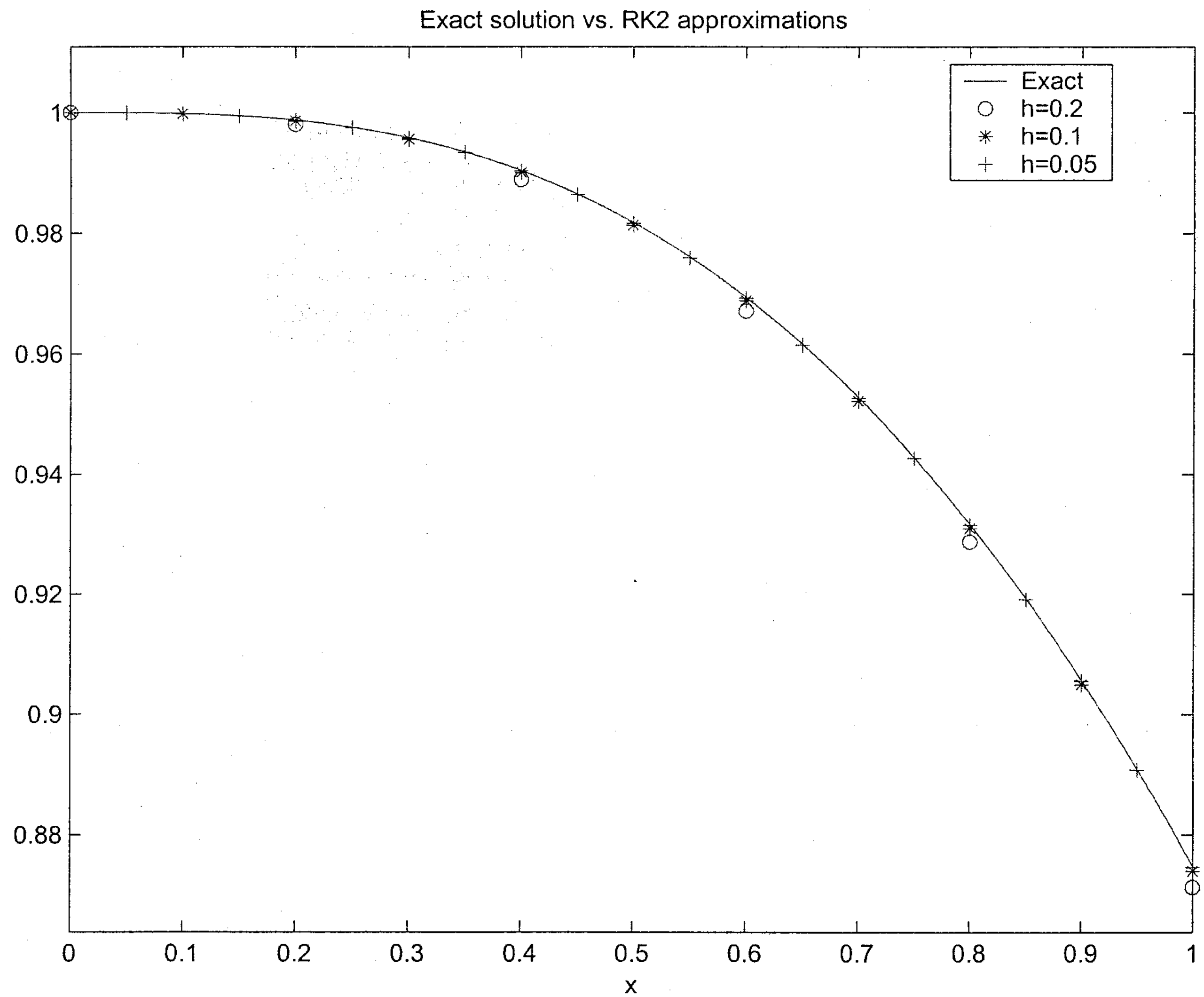
$\boxed{5/6.2}$ See table at the end of the book

$\boxed{7/6.2}$ (ii) Solve the linear equation

$$z' = -z + \cos x \ , \quad z(0) = 1$$

and get $z(x) = \frac{1}{2}\left(\cos x + \sin x + e^{-x}\right)$

(i), (iii) See graph



$\boxed{9/6.3}$ $x' = \frac{\sin t}{x+1}$ $x(0)=0.$

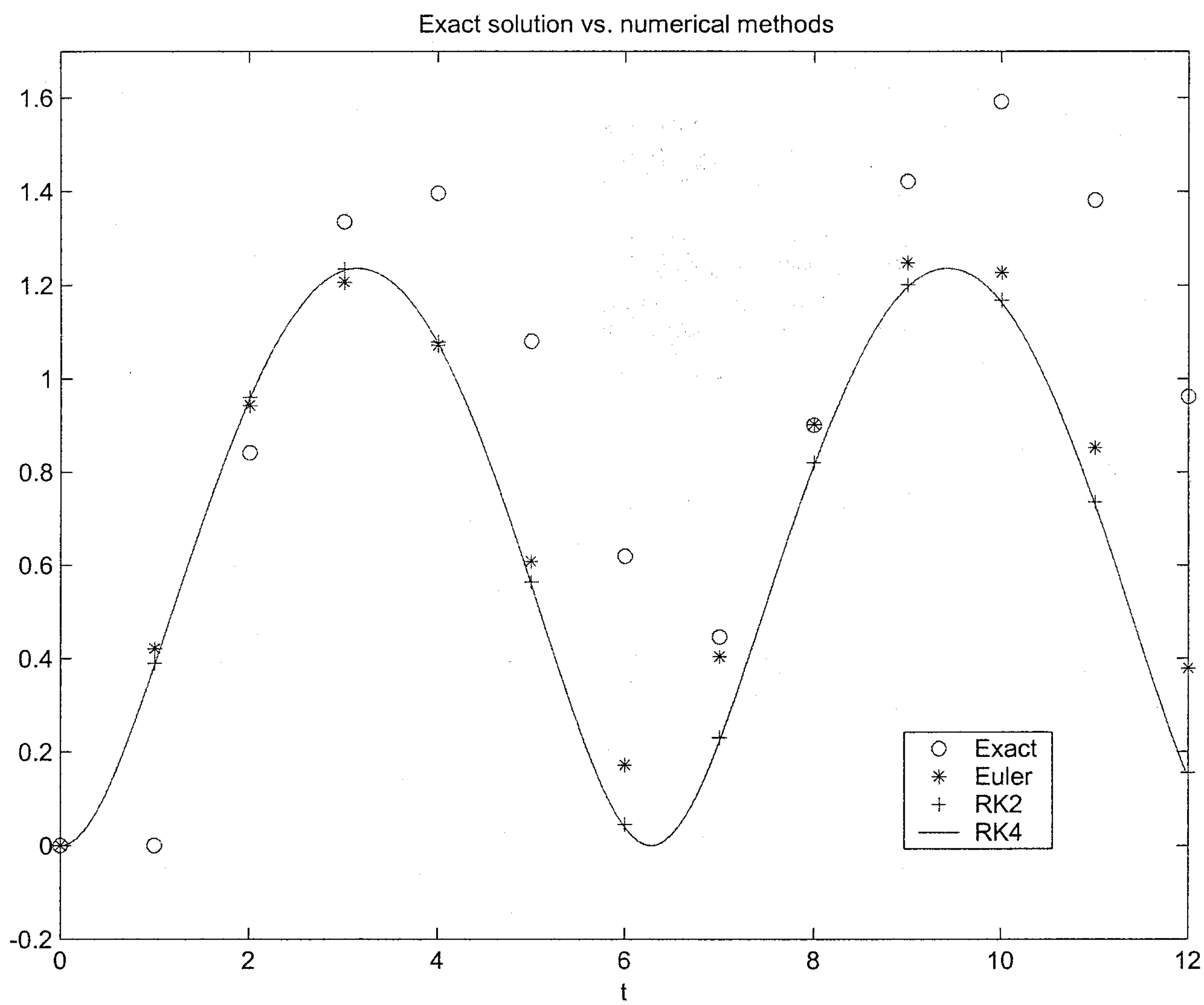
Separable equation: $\int (x+1)\,dx = \int \sin t\,dt$

$\frac{1}{2}x^2 + x = -\cos t + C.$ Since $x(0)=0 \Rightarrow C=1$

Solve: $\frac{1}{2}x^2 + x + (\cos t - 1) = 0$

$x_{1,2} = -1 \pm \sqrt{1-2\cos t+2}$. Since $x(0)=0$

we get $x(t) = -1 + \sqrt{3-2\cos t}$.



$\boxed{11/7.1}$ By a direct computation, check that

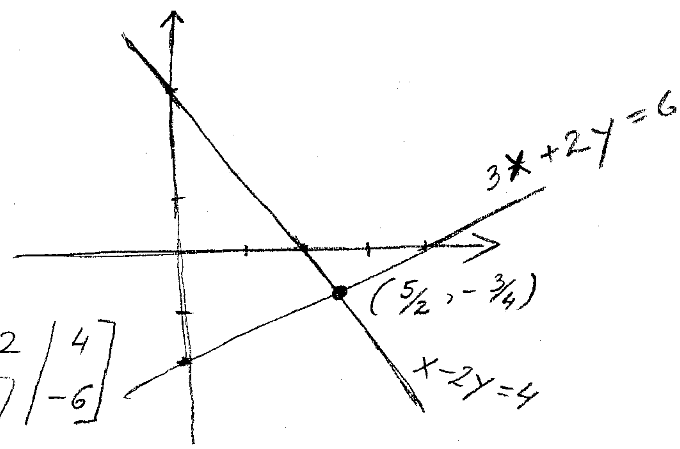
$$A(a\bar{x}) = aA\bar{x} \text{ and } A(\bar{x}+\bar{y}) = A\bar{x} + A\bar{y}.$$

35/7.1.
$$\begin{bmatrix} -6 & -1 & 7 \\ -1 & 8 & -9 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -6\cdot(-1) + (-1)\cdot 0 + 7\cdot 2 \\ -1\cdot(-1) + 8\cdot 0 + (-9)\cdot 2 \end{bmatrix} = \begin{bmatrix} 20 \\ -17 \end{bmatrix}$$

46/7.1
$$D^T = \begin{bmatrix} 10 & -5 & 3 \\ 0 & 8 & 6 \\ 0 & -9 & 7 \\ -1 & 3 & 6 \end{bmatrix}$$

$\boxed{53/7.1}$
$$\begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

$\boxed{6/7.2}$
$$x - 2y = 4$$
$$3x + 2y = 6$$



(i) See graph

(ii) $\left[\begin{array}{cc|c} \boxed{1} & -2 & 4 \\ 3 & 2 & 6 \end{array}\right] \underset{\text{to } R_2}{\overset{\text{add } -3R_1}{\sim}} \left[\begin{array}{cc|c} \boxed{1} & -2 & 4 \\ 0 & \boxed{8} & -6 \end{array}\right]$

so $8y = -6 \Rightarrow y = -\frac{3}{4}$

$x - 2y = 4 \Rightarrow x = 4 + 2y = 4 - \frac{3}{2} = \frac{5}{2}$

$\boxed{17/7.2}$
$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & -3 & 6 \\ 2 & 5 & 8 & 40 \end{array}\right] \underset{\text{to } R2}{\overset{\text{add } -2R_1}{\sim}} \left[\begin{array}{ccc|c} \boxed{1} & 2 & -3 & 6 \\ 0 & \boxed{1} & 14 & 28 \end{array}\right]$$

$x + 2y - 3z = 6$

$y + 14z = 28$

z - free $(=t)$

Then $y = 28 - 14t$

$x = 6 + 3t - 2(28 - 14t)$

$= -50 + 31t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -50 \\ 28 \\ 0 \end{bmatrix} + t\begin{bmatrix} 31 \\ -14 \\ 1 \end{bmatrix}$$

$\boxed{27.}$ Notice that S is just a half-line, so it cannot be the solution set of a linear system

31. $$\bar{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \end{bmatrix} \quad (L)$$

Let's look for a possible system of linear equations.

Since $y_1 = 2t$ then $t = \frac{y_1}{2}$ and plug-in
$y_2 = 1 - 3t$

$$y_2 = 1 - 3t = 1 - 3 \cdot \frac{y_1}{2}, \text{ or } \boxed{\frac{3}{2} y_1 + y_2 = 1.}$$

This is the equation that has L as solution set.

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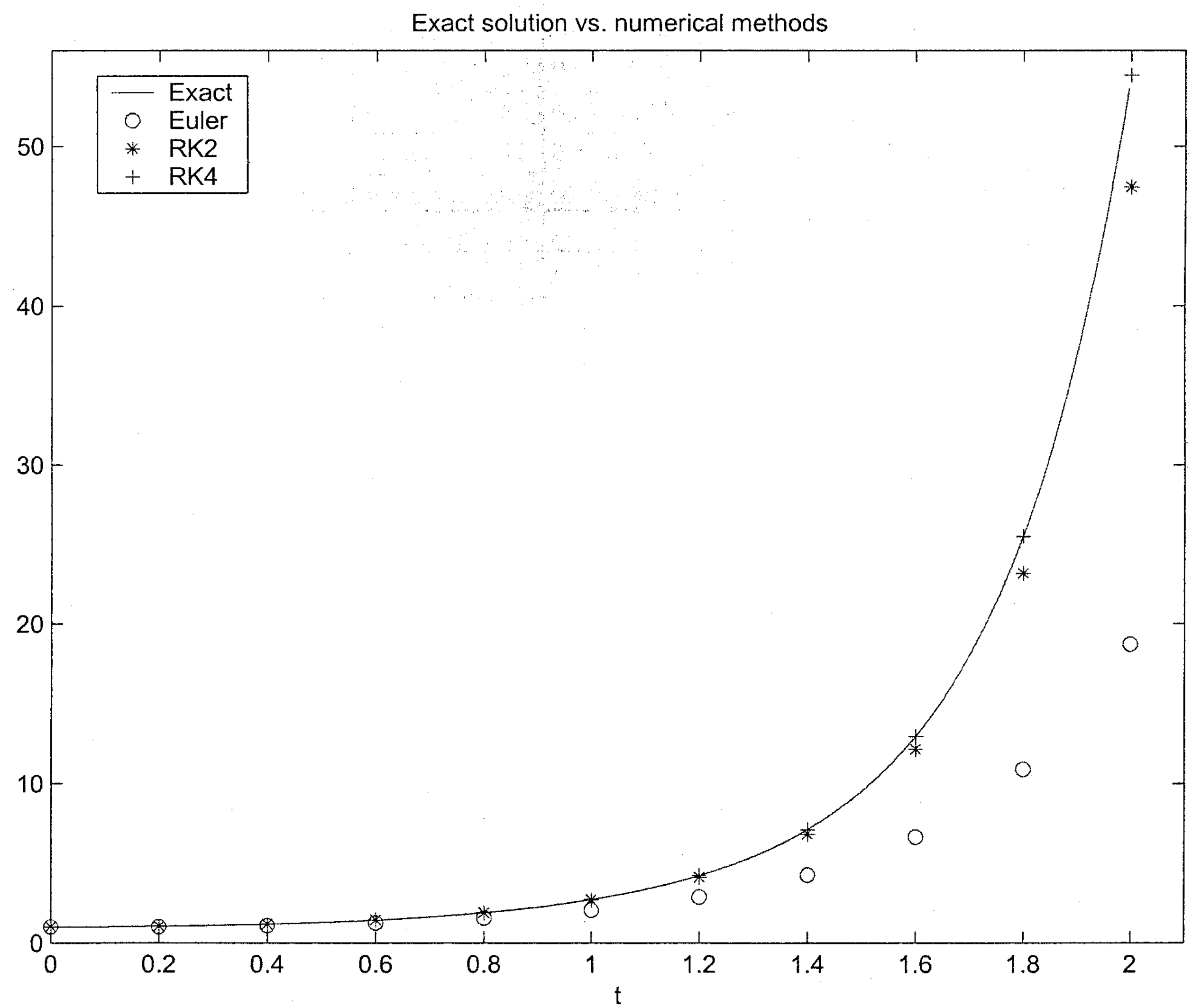
(a) $y' = 2ty \qquad y(0) = 1$

Exact solution $y(t) = e^{t^2}$

(b) See graph



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Exact solution: $y' = \frac{2}{t} y + 1 \quad y(1) = 1$

Linear equation, so $y(t) = 2t^2 - t$

See graph below for a log log plot of Max. error vs. step size for the three methods.