Math 211 - Hw #7

1/7.5 Need to check whether there exist c_1, c_2 so that

$$c_1 \bar{u}_1 + c_2 \bar{u}_2 = \bar{w}$$

This is equivalent to solving the system

$$\left[\begin{array}{cc|c} \boxed{1} & 3 & 5 \\ -2 & 0 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 6 & 8 \end{array}\right] \quad \text{so}$$

$$\begin{aligned} c_1 + 3c_2 &= 5 \\ 6c_2 &= 8 \end{aligned} \Rightarrow \quad c_2 = \frac{4}{3}, \quad c_1 = 5 - 3\cdot\frac{4}{3} = 1$$

The vector $\bar{w} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ is in the span$\{\bar{u}_1, \bar{u}_2\}$ because

$$\bar{w} = 1\cdot \bar{u}_1 + \frac{4}{3}\bar{u}_2$$

5/7.5 Need to solve the system

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -4 & -2 & 4 \\ 4 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & 8 \\ 0 & -1 & -3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{array}\right]$$

which is inconsistent. Hence $\bar{w} \notin \text{span}\{\bar{v}_1, \bar{v}_3\}$

10/7.5 Need to solve the system $c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3 = \bar{w}$, or

$$\left[\begin{array}{ccc|c} 0 & -2 & -2 & w_1 \\ 1 & 1 & -2 & w_2 \\ -2 & -4 & 0 & w_3 \end{array}\right] \sim \dots \sim \left[\begin{array}{ccc|c} 1 & 1 & -2 & -w_2 \\ 0 & 1 & 1 & -\frac{1}{2}w_1 \\ 0 & 0 & 10 & -3w_1 - 2w_2 + w_3 \end{array}\right]$$

This is a consistent system, so $\bar{w} \in \text{span}\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$

Moreover, $c_1 + c_2 - 2c_3 = -w_2$

$$c_2 + c_3 = -\frac{1}{2}w_1 \qquad \Rightarrow \quad c_3 = \frac{-3w_1 - 2w_2 + w_3}{10}$$

$$10c_3 = -3w_1 - 2w_2 + w_3 \qquad c_2 = \frac{-2w_1 + 2w_2 - w_3}{10}$$

$$c_1 = \frac{4w_1 - 4w_2 - 3w_3}{10}$$

29/7.5. Need to solve $A\bar{x} = \bar{0}$ in order to obtain a basis for null(A).

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -5 & -2 & -5 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So $x_1 + x_2 + x_3 = 0$
$x_2 = 0$
x_3 - free

$$\Rightarrow \bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

So Null$(A) = \left\{ t \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} : t \text{ - free parameter} \right\}$ and a basis is given by $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

11/7.7.

$$\begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & 2 & -2 & 0 \\ -1 & 2 & 0 & 0 \\ -1 & 3 & 1 & 2 \end{vmatrix} \overset{R_1 \leftrightarrow R_3}{=} - \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 2 & -1 & 3 & 4 \\ -1 & 3 & 1 & 2 \end{vmatrix} =$$

$$= - \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 3 & 3 & 4 \\ 0 & 1 & 1 & 2 \end{vmatrix} \overset{R_2 \leftrightarrow R_4}{=} + \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 3 & 3 & 4 \\ 0 & 2 & -2 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -4 & -4 \end{vmatrix}$$

$$\overset{R_3 \leftrightarrow R_4}{=} - \begin{vmatrix} -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -4 & -4 \\ 0 & 0 & 0 & -2 \end{vmatrix} = -(-1)\cdot(1)\cdot(-4)\cdot(-2) = 8.$$

20/7.7 Expand by row 3 (easiest)

$$\det A = \begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & 2 & -2 & 0 \\ -1 & 2 & 0 & 0 \\ -1 & 3 & 1 & 2 \end{vmatrix} = (-1)^{3+1}\cdot(-1)\cdot\begin{vmatrix} -1 & 3 & 4 \\ 2 & -2 & 0 \\ 3 & 1 & 2 \end{vmatrix} +$$

$$+ (-1)^{3+2}\cdot(2)\cdot\begin{vmatrix} 2 & 3 & 4 \\ 0 & -2 & 0 \\ -1 & 1 & 2 \end{vmatrix}$$

$$= -24 - 2\cdot(-16) = 8.$$

37/7.7 The nullspace is nontrivial $\Leftrightarrow \det A = 0$

$$\begin{vmatrix} 2-x & 0 & 0 \\ -1 & -x & 2 \\ 0 & -2 & 5-x \end{vmatrix} = (2-x)\cdot(-x)(5-x) + 4\,(2-x)$$
$$= (2-x)(x^2-5x+4) = (2-x)(x-1)(x-4).$$

So $\det A = 0$ implies $x = 1, 2, 4$.

50/7.7 (a) $\det(-2U) = (-2)\cdot(-2)\cdot(-2)\cdot(-2)\cdot\det(U)$

$= 16\det(U) = -48.$

(b) $\det(U^3) = [\det(U)]^3 = (-3)^3 = -27$

(c) $\det(U^{-1}) = \dfrac{1}{\det U} = -\dfrac{1}{3}.$

9/8.1. Notice that $x'(t) = -e^{-t}(-\cos t - \sin t) + e^{-t}(\sin t - \cos t)$
$$= 2e^{-t}\sin t = v(t)$$
Also $v'(t) = -2 e^{-t}\sin t + 2e^{-t}\cos t = -2x(t) - 2v(t)$

Moreover $x(0) = -1$ and $v(0) = 0$.

11/8.1. Let $x_1(t) = y(t)$ and $x_2(t) = y'(t)$.

Then $y'' + 2y' + 4y = 3\cos 2t$ becomes
$$x_1' = y' = x_2$$
$$x_2' = y'' = 3\cos 2t - 2y' - 4y$$
$$= -4x_1 - 2x_2 + 3\cos 2t$$

So
$$\bar{x}' = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} x_2 \\ -4x_1 - 2x_2 + 3\cos 2t \end{bmatrix}; \quad \bar{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

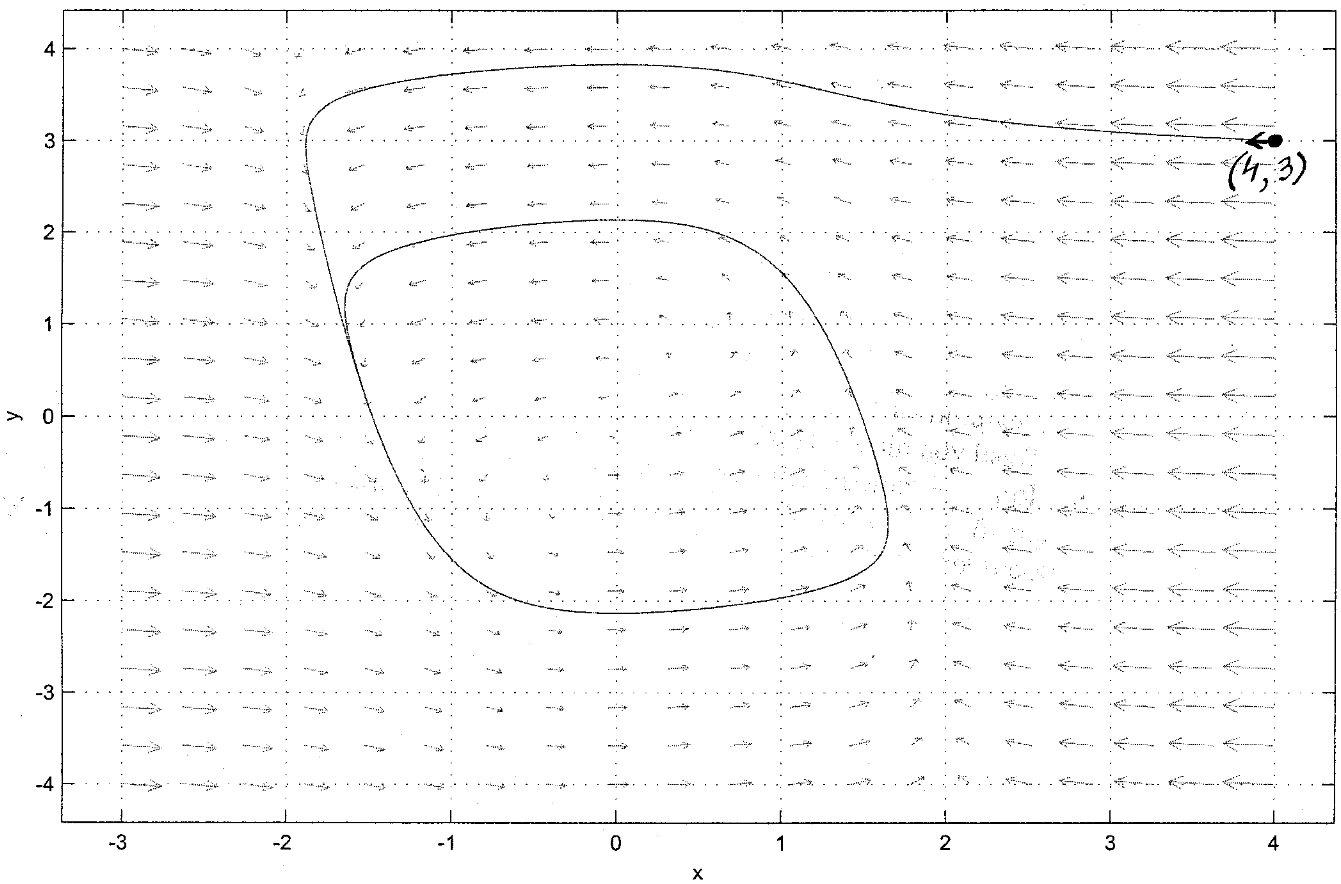
19. See plots

26. (a) The orbits in the phase plane are closed loops, so we have periodic behavior.

(b) The point $(3,2)$ is a fixed point for the system (equilibrium solution).

19. (b)

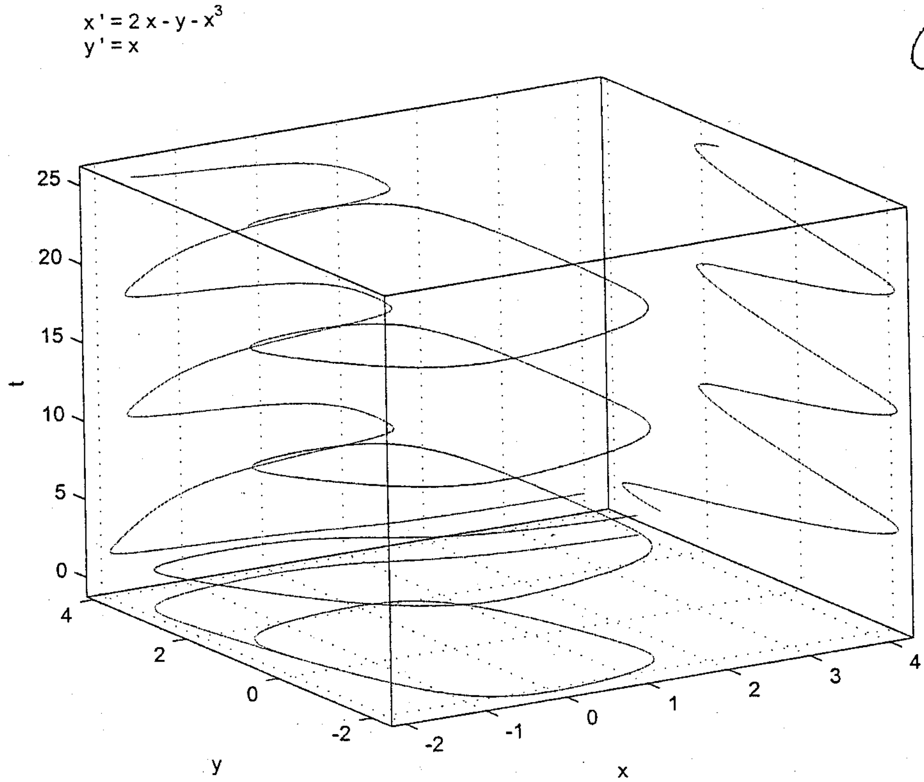
$x' = 2x - y - x^3$
$y' = x$



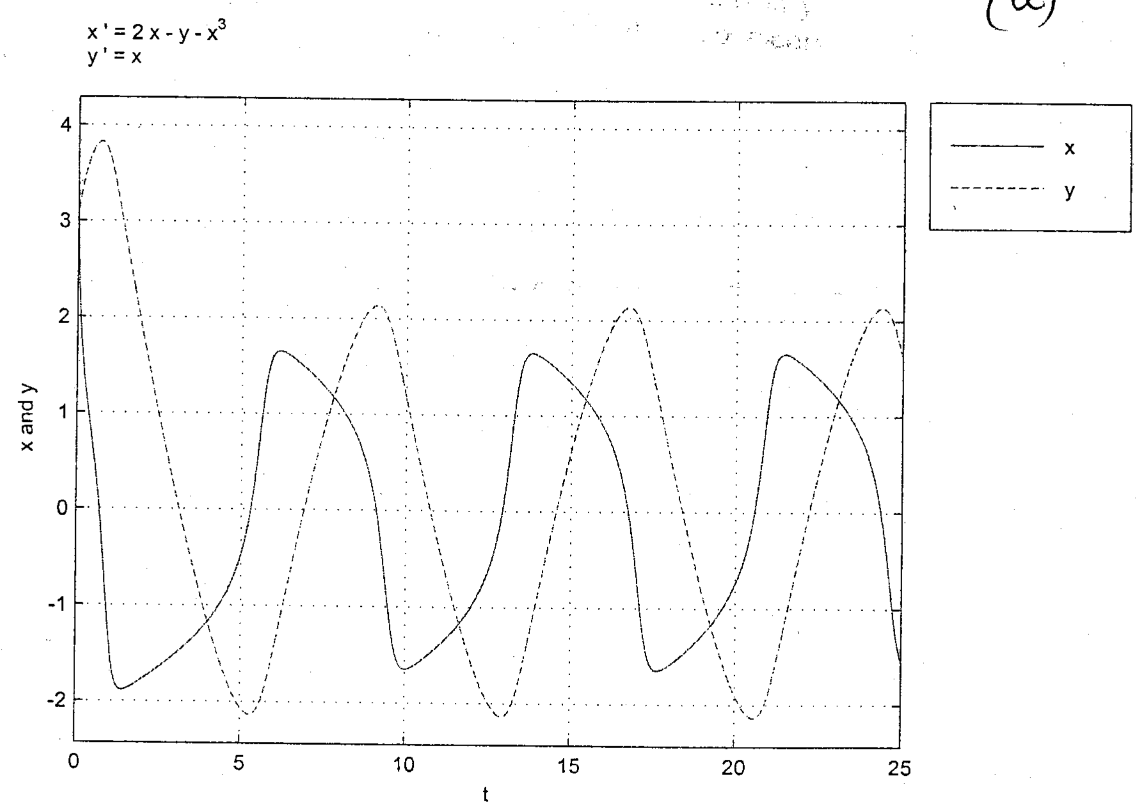
19.

Composite plot

(c)



(a)



26.