MATH 213 - Homework #1 (Due 1/20)

1. A biologist prepares a culture. After 1 day of growth, the biologist counts 1000 cells. After 2 days of growth, he counts 3000. Assuming a Malthusian model, what is the reproduction rate and how many cells were present initially? What will be the number of cells after 4 days?
2. Solve problem #2 on page 152.
3. Solve problem #3 on page 152.
4. A population is observed to obey the logistic equation with carrying capacity 20,000. The initial population is 1000, and 8 hours later, the observed population is 1200. Find the natural reproductive rate and the time required for the population to reach 75% of its carrying capacity.
5. Assume that the rate of change for a population is given by the differential equation
$$\frac{dN}{dt} = rN^2 \,.$$
where $r > 0$, and $N(0) = N_0$.
   (a) Find the explicit solution of this initial value problem.
   (b) Plot a representative graph for such solutions.
   (c) Show that the solution becomes infinite in finite time, and determine the maximal interval of existence for such a solution.
6. A relatively simple equation modeling the growth of solid tumors is given by the Gompertz growth law
$$\frac{dN}{dt} = re^{-at}N$$
where $N(t)$ denotes the tumor cell population at time t.
   (a) Give a reason for considering the growth rate to be re^{-at}.
   (b) Find an explicit solution assuming $r = 10$, $a = 1$, and $N(0) = 1$. What is the long term behavior of the tumor growth?
   (c) Find the solution in the general situation (assuming r, a arbitrary parameters and $N(0) = N_0$).