

MATH 213 - Homework #2 (Due 1/27)

1. Use the *dsolve* command in MATLAB to find the solution of the following initial value problems. Then use the *ezplot* command to plot the solution over the indicated interval.

(a) $y' = -2ty, y(0) = 1, [-2, 2]$; (b) $y' + 2y = \cos(t), y(0) = 1, [0, 20]$.

2. Use *dfield* to plot a few solution curves for the equations:

(a) $y' = 2ty/(1 + y^2)$ with a display window $t \in (-3, 10), y \in (-5, 5)$.

(b) $y' = 1 - t^2 + \sin(ty)$ with a display window you consider appropriate.

3. Perform a qualitative analysis (equilibrium solutions, phase line, stability, solutions sketch) for the following autonomous equations:

(a) $y' = (y + 1)(y - 4)$; (b) $y' = 9y - y^3$.

4. Consider a fish population governed by the logistic equation

$$N' = 0.1N(1 - N/10)$$

where time is measured in days and N in thousands of fish. Suppose that fishing is started in this lake and that 100 fish are removed each day.

- (a) Modify the logistic equation to account for the fishing.
 - (b) Find and classify the equilibrium solutions for your model.
 - (c) Use qualitative analysis to discuss the fate of the fish population. In particular, if the initial fish population is 1000, what happens to the fish as time passes? What will happen to an initial population having 2000 fish?
5. Consider the logistic equation $N' = rN(1 - N/K)$ where the carrying capacity is time-dependent. In each of the following cases, study the long term behavior of $N(t)$, using *dfield*.
 - (a) Assume $K(t) = 1 - \frac{1}{2}e^{-t}$, $r = 1$. This might model a human population where, due to technological improvement, the availability of resources is increasing with time, although ultimately limited.
 - (b) Assume $K(t) = 1 - \frac{1}{2}\cos(2\pi t)$, $r = 1$. Here the carrying capacity is periodic in time with period 1. This models, for example, a population of insects or small animals affected by the seasons.