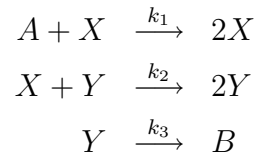


MATH 213 - Homework #8 (Due 4/19)

1. Consider a SIRS model with vital dynamics, i.e. the population is assumed to be producing new susceptibles at a rate δ which is identical to the mortality rate (see Figure 6.12c, page 244) .
 - (a) Write down the system of differential equations modeling this situation and show that the total population $N = S + I + R$ is constant.
 - (b) Find the steady states.
 - (c) Using the relation $R = N - S - I$ reduce the SIRS system to a planar SI system and find the steady states for it.
 - (d) Perform a qualitative analysis (using also pplane) for the case $\beta = 0.2$, $\gamma = 0.2$, $\nu = 0.3$, $\delta = 0.1$.
 - (e) Perform a qualitative analysis (using also pplane) for the case $\beta = 0.6$, $\gamma = 0.2$, $\nu = 0.2$, $\delta = 0.1$.
2. Problem 29, page 263.
3. Problem 8, page 302.
4. The following chemical reaction mechanism was studied by Lotka in 1920, and later in 1956:



Assume that A and B are kept at a constant concentration.

- (a) Using the law of mass action, show that the concentrations $x(t)$ and $y(t)$ of substances X and Y satisfy the following equations:

$$\begin{aligned}x' &= k_1Ax - k_2xy \\y' &= k_2xy - k_3y\end{aligned}$$

- (b) Find the steady states and their type (if possible). Use pplane to illustrate the long-term behavior of this system.