MATH 213 - Homework #9 (Due 4/28)

1. Solve the following difference equations subject to the specified initial conditions:
   (a) $x_{n+2} = 5x_{n+1} + 6x_n$, $x_0 = 2$, $x_1 = 5$;
   (b) $x_{n+2} = x_{n+1} - x_n$, $x_0 = 1$, $x_1 = 2$.

2. Find the general solution (vector-form) for the following linear discrete systems:
   (a) $x_{n+1} = \dfrac{x_n}{4} + y_n$, $y_{n+1} = \dfrac{3x_n}{16} - \dfrac{y_n}{4}$;
   (b) $x_{n+1} = x_n - y_n$, $y_{n+1} = 6x_n - 3y_n$.

3. Consider the Red Blood Cell Population Model described on page 27. In class we analyzed the case $\gamma = 1$. Prove that
   (a) If $\gamma < 1$, then the number of RBCs $R_n \to 0$ as $n \to \infty$.
   (b) If $\gamma > 1$, then the number of RBCs $R_n \to \infty$ as $n \to \infty$.

   (HINT: The eigenvalues are $\lambda_{1,2} = \dfrac{(1-f) \pm \sqrt{(1-f)^2 + 4\gamma f}}{2}$)

4. Determine the steady states and their stability for the following nonlinear difference equations defined on the interval $[0, \infty)$:
   (a) $x_{n+1} = \dfrac{1}{2 + x_n}$.
   (b) $x_{n+1} = \dfrac{2x_n}{1 + x_n}$.

   Use the cobwebbing method to sketch the approximate behavior of solutions from some initial starting value of x_0.

5. Problem #3, page 62.