**Math 451 HW 1, due Friday 9/2/22 NAME:**

1. Consider spherical coordinates on $\mathbb{R}^3$ (not including the line $x = y = 0$) $\rho, \phi, \theta$ defined in terms of the Euclidean coordinates $x, y, z$ by

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi.$$

   (a) Express $\partial/\partial\rho$, $\partial/\partial\phi$, and $\partial/\partial\theta$ as linear combinations of $\partial/\partial x$, $\partial/\partial y$, and $\partial/\partial z$. (The coefficients in these linear combinations will be functions on $\mathbb{R}^3 \setminus \{x = y = 0\}$.)

   (b) Express $d\rho$, $d\phi$, and $d\theta$ as linear combinations of $dx$, $dy$, and $dz$.

2. Lee 1-8 [Second Edition]
By identifying $\mathbb{R}^2$ with $\mathbb{C}$, we can think of the unit circle $S^1$ as a subset of the complex plane. An angle function on a subset $U \subset S^1$ is a continuous function $\theta : U \to \mathbb{R}$ such that $e^{i\theta(z)} = z$ for all $z \in U$. Show that there exists an angle function on an open subset $U \subset S^1$ if and only if $U \neq S^1$. For any such angle function, show that $(U, \theta)$ is a smooth coordinate chart for $S^1$ with its standard smooth structure.

3. Lee Proposition 2.10, Exercise 2.11 [Second Edition]
Let $M, N, P$ be smooth manifolds without boundary. Prove the following:

   (a) Every constant map $c : M \to N$ is smooth.

   (b) The identity map of $M$ is smooth.

   (c) If $U \subset M$ is an open submanifold without boundary, then the inclusion map $i : U \hookrightarrow M$ is smooth. *(See Exercise 1.44 for the definition of open submanifold without boundary.)*

   *To see a flavor of how these arguments go, read the proof of (d) in Lee.*

4. Please spend some time reading Chapters 1-3 of Lee. Remember that you'll want the angle function in a future homework set to show that $TS^1$ is diffeomorphic to $S^1 \times \mathbb{R}$. In Chapter 1, it's not absolutely necessary to read about manifolds with boundary. I'll revisit these notions when we get to integration on manifolds. In Chapter 2, you can omit as much of the reading about partitions of unity as you would like. We won't use partitions of unity until we talk about integration and Riemannian metrics (at least 1 month away). In Chapter 2, the book alludes to the difficulties in distinguishing between smooth structures up to diffeomorphism. This is a subtle point I hope to explain more about as the semester progresses.

* Which problems provided a worthwhile learning experience? How many hours did you spend on it?