Updated: Math 451 HW #2, due Friday 9/9/22 **NAME:**

1. Lee 3-4 [Second Edition]
   Show that TS^1 is diffeomorphic to $S^1 \times \mathbb{R}$.

2. **Updated** Lee 3-1 [Second Edition]
   Suppose M and N are smooth manifolds without boundary and $F : M \to N$ is a smooth map. Show that $dF_p : T_pM \to T_{F(p)}N$ is the zero map for each $p \in M$ if and only if F is constant on each component.

3. Show that if M and N are smooth manifolds and if $p \in M$ and $q \in N$, then there is a canonical isomorphism

$$T_{(p,q)}(M \times N) = T_pM \oplus T_qN.$$

   It is painful to describe this isomorphism in full detail with respect to derivations or linear combinations of partial derivatives with respect to coordinate charts. (Maybe think about what is happening if you are so inclined.)

   Set up to show there is a canonical isomorphism:
   Let $(x, y) \in M \times N$. Consider the canonical inclusion maps $i^M : x \mapsto (x, q)$ and $i^N : y \mapsto (p, y)$ as well as the projection maps $\pi^M : (x, y) \mapsto x$ and $\pi^N : (x, y) \mapsto y$.

   Define $\Phi : T_pM \oplus T_qN \to T_{(p,q)}(M \times N)$ by

$$\Phi(v, w) = di^M_p(v) + di^N_p(w)$$

   and define $\Psi : T_{(p,q)}(M \times N) \to T_pM \oplus T_qN$ by

$$\Psi(v) = (d\pi^M_{(p,q)}(v), d\pi^M_{(p,q)}(w)).$$

   Deduce that $\Psi \circ \Phi$ is the identity on $T_pM \oplus T_qN$. (This can be done by showing and then using the fact that $d\pi^N \circ di^M = 0$ and $d\pi^M \circ di^N = 0$.). Now use linear algebra facts to conclude that Ψ and Φ are isomorphisms.

* Which problems provided a worthwhile learning experience? How many hours did you spend on it?