Math 4081 HW#3, due Wednesday 2/14/18 **NAME:**

1. The *zero section* of the tangent bundle TM is the set of zero tangent vectors,

$$Z = \{(p, 0)\} \subset TM = \{(p, V) \mid p \in M, V \in T_pM\}.$$

   (a) Show that Z is a submanifold of TM which is diffeomorphic to M.

   (b) Show that if $(p, 0) \in Z$, then there is a canonical (not depending on a choice of coordinates) isomorphism

$$T_{(p,0)}TM = T_pM \oplus T_pM.$$

2. The *Hopf fibration* is the map $f : S^3 \to \mathbb{C}P^1$ sending $(z^1, z^2) \in \mathbb{C}^2$ with $|z^1|^2 + |z^2|^2 = 1$ to $[z^1 : z^2] \in \mathbb{C}P^1$. Show that the Hopf fibration is a submersion.

3. (Lee Second 4-6) Let M be a nonempty smooth compact manifold. Show that there is no smooth submersion $F : M \to \mathbb{R}^k$ for any $k > 0$.

4. (Lee Second 5-1) Consider the map $\Phi : \mathbb{R}^4 \to \mathbb{R}^4$ defined by

$$\Phi(x, y, s, t) = (x^2 + y, x^2 + y^2 + s^2 + t^2 + y).$$

Show that $(0, 1)$ is a regular value of ϕ and that the level set $\Phi^{-1}(0, 1)$ is diffeomorphic to S^2.

5. For which values of R does the hyperboloid defined by $x^2 + y^2 - z^2 = 1$ intersect the sphere $x^2 + y^2 + z^2 = R$ transversely? What does the intersection look like for different values of R?

6. (Lee Second 6-9) Let $F : \mathbb{R}^2 \to \mathbb{R}^3$ be the map defined by

$$F(x, y) = (e^y \cos x, e^y \sin x, e^{-y}).$$

a) For which positive numbers r is F transverse to the 2-sphere of radius r, $S_r(0) \subset \mathbb{R}^3$?
b) For which positive numbers r is $F^{-1}(S_r(0))$ an embedded submanifold of $\mathbb{R}^2$?

7. How difficult was this assignment?