Math 4081 HW#5, due Monday 3/26/18 NAME:

1. Lee SECOND 9-3. (8 points)
   Compute the flow of each of the following vector fields on $\mathbb{R}^2$:

   (a) $V = y\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y}$

   (b) $W = x\dfrac{\partial}{\partial x} + 2y\dfrac{\partial}{\partial y}$

   (c) $X = x\dfrac{\partial}{\partial x} - y\dfrac{\partial}{\partial y}$

   (d) $Y = x\dfrac{\partial}{\partial y} + y\dfrac{\partial}{\partial x}$

2. Lee SECOND 9-16. (3 points)
Give an example of smooth vector fields $V, \widetilde{V}, and W$ on $\mathbb{R}^2$ such that

$$V = \widetilde{V} = \frac{\partial}{\partial x}$$

along the x-axis but

$$\mathcal{L}_V W \neq \mathcal{L}_{\widetilde{V}} W$$

at the origin. Remark: this shows that it is really necessary to know the vector field V to compute $(\mathcal{L}_V W)_p$; it is not sufficient just to know the vector V_p, or even to know the values of V along an integral curve of V.

3. Lee SECOND 9-17. (6 points)
For each k-tuple of vector fields on $\mathbb{R}^3$ shown below, either find smooth coordinates (s^1, s^2, s^3) in a neighborhood of $(1, 0, 0)$ such that $V_i = \dfrac{\partial}{\partial s^i}$ for $i = 1, ..., k$ or explain why there are none.

   (a) $(k = 2)$; $V_1 = \dfrac{\partial}{\partial x}, \quad V_2 = \dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y}$

   (b) $(k = 2)$; $V_1 = (x+1)\dfrac{\partial}{\partial x} - (y+1)\dfrac{\partial}{\partial y}, \quad V_2 = (x+1)\dfrac{\partial}{\partial x} + (y+1)\dfrac{\partial}{\partial y}$

   (c) $(k = 3)$; $V_1 = x\dfrac{\partial}{\partial y} - y\dfrac{\partial}{\partial x}, \quad V_2 = y\dfrac{\partial}{\partial z} - z\dfrac{\partial}{\partial y}, \quad V_3 = z\dfrac{\partial}{\partial x} - x\dfrac{\partial}{\partial z}$

4. Lee SECOND 9-18. (3 points)
Define vector fields X and Y on the plane as in (1) by

$$X = x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y} \qquad Y = x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x}$$

Recall that you computed the flows ϕ and ψ of X and Y. Now verify that the flows do not commute by finding explicit open intervals I and J containing 0 such that $\phi_s \circ \psi_t$ and $\psi_t \circ \phi_s$ are both defined for all $(s,t) \in I \times J$, but they are unequal for some such (s,t).

5. How difficult was this assignment?