Math 211-003: Assignment 10

Due 12/04/2008

These problems are due with work shown by the beginning of class.

1. There are two tanks. The first tank initially has 10 gallons of pure water. The second tank initially has 8 gallons of a water/salt solution with 10 oz of water. Both tanks drain into the other at a rate of 2 gallons per minute. Find formulas to express the amount of salt in each tank.
   To review systems, look at Sec 9.2)#7,9,11,17,19,21,41,43,45,47,49,51,53,55,58,59

2. Solve the following IVP:
$$\vec{x}' = \begin{pmatrix} -1 & -4 \\ -2 & 1 \end{pmatrix} \vec{x} \text{ for } \vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

3. Find a general solution to the following system
$$\vec{x}' = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix} \vec{x}$$

4. Solve the following IVP:
$$\vec{x}' = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \vec{x} \text{ for } \vec{x}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

5. Find a general solution to the following system
$$\vec{x}' = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
   To review inhomogeneous system, Sec 9.9)#1-6

6. Given an $n \times n$ matrix A, show that the set
$$\mathbf{E}_\lambda := \{\vec{v} | A\vec{v} = \lambda\vec{v}\}$$
   is a vector subspace whenever $\mathbf{E}_\lambda \neq \{\vec{0}\}$.
   See the definition of a vector subspace on p.308

7. Find the values of α such that the system
$$\begin{pmatrix} \alpha & 1 & 0 \\ 1 & \alpha & 1 \\ 0 & 1 & \alpha \end{pmatrix} \vec{x} = \vec{b}$$
   is guaranteed to have a solution for any choice of $\vec{b}$.
   See Sec 7.7)#30-39

8. Determine which of the following vectors are in V, where

$$V = span\left\{\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}\right\}$$

$$\vec{v_1} = \begin{pmatrix}1\\-1\\2\end{pmatrix}, \vec{v_2} = \begin{pmatrix}1\\0\\1\\0\end{pmatrix}, \vec{v_3} = \begin{pmatrix}0\\0\\1\end{pmatrix}, \vec{v_4} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

9. Determine which of the following vectors are in V, where

$$V = span\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$$

$$\vec{v_1} = \begin{pmatrix}1\\2\\3\end{pmatrix}, \vec{v_2} = \begin{pmatrix}e^2\\\pi\\\cos 1\end{pmatrix}, \vec{v_3} = \begin{pmatrix}1\\1\\1\end{pmatrix}, \vec{v_4} = \begin{pmatrix}5\\4\\\frac{7}{2}\end{pmatrix}$$

See 7.6)#1-10

10. Find a basis for

$$V = span\left\{\begin{pmatrix}1\\2\\3\\4\end{pmatrix}, \begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\2\\4\\6\end{pmatrix}, \begin{pmatrix}2\\0\\-2\\-4\end{pmatrix}\right\}$$

See 7.5)#33-40

11.

$$A = \begin{pmatrix}2 & 0 & 1 & 1\\0 & 0 & 1 & -1\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{pmatrix}$$

a) What is the dimension of $null(A)$.
b) Find a basis for $null(A)$.
c) Find the general solution for

$$A\vec{x} = \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$$

See 7.4)#18-21 and 7.5)#25-32

12. Solve the following IVP:

$$x' = x \cos t \text{ for } x(0) = 5$$

13. Find a general solution for

$$x'' - 3x' + 2x = \sin t$$

14. Suppose $(\lambda, \vec{v})$ is an eigenpair for an $n \times n$ matrix A. Suppose also that $(\mu, \vec{v})$ is an eigenpair for $n \times n$ matrix B. Show that $(e^{\lambda+\mu}, \vec{v})$ is an eigenpair for e^{A+B}
    See 9.6)#5,6,7,8,9,10,13