Math 211-003: Assignment 2
Due 9/11/2008

These problems are due with work shown by the beginning of class.

#1) Sec 2.5, #2,6 *(switch the in and out volume rates so the tank is draining), 12

We are going to prove a theorem known as Grönwall’s Inequality, stated as follows:

Over an interval \([0, r]\) for positive \(r\), suppose we have a differentiable function \(x : [0, r) \rightarrow \mathbb{R}\) such that

\[
\frac{d}{dt} x \leq a(t)x(t), \forall t \in [0, r) \text{ and } x(0) = x_0
\]

Then

\[
x(t) \leq x_0 e^{\int_0^t a(u) du}, \forall t \in [0, r)
\]

#2) Find a function \(v(t)\) defined on (at least) \([0, r)\) that satisfy the following conditions:

- \(v' = av\)
- \(v > 0\)
- \(v(0) = 1\)

(*HINT*)the answer is in the statement of the theorem

#3) Show that

\[
\frac{d}{dt} \left( \frac{x}{v} \right) \leq 0
\]

#4) Use #3 to conclude that over \([0, r)\),

\[
\frac{x}{v} \leq x_0 \text{ or equivalently } x(t) \leq x_0 v(t)
\]