

Math 211-003: Assignment 2

Due 9/11/2008

These problems are due with work shown by the beginning of class.

#1) Sec 2.5, #2,6 *(switch the in and out volume rates so the tank **is** draining),12

We are going to prove a theorem known as **Grönwall's Inequality**, stated as follows:

Over an interval $[0, r)$ for positive r , suppose we have a differentiable function $x : [0, r) \rightarrow \mathbb{R}$ such that

$$\frac{d}{dt}x \leq a(t)x(t), \forall t \in [0, r) \text{ and } x(0) = x_0$$

Then

$$x(t) \leq x_0 e^{\int_0^t a(u)du}, \forall t \in [0, r)$$

#2) Find a function $v(t)$ defined on (at least) $[0, r)$ that satisfy the following conditions:

- $v' = av$
- $v > 0$
- $v(0) = 1$

(HINT) the answer is in the statement of the theorem

#3) Show that

$$\frac{d}{dt} \left(\frac{x}{v} \right) \leq 0$$

#4) Use #3 to conclude that over $[0, r)$,

$$\frac{x}{v} \leq x_0 \text{ or equivalently } x(t) \leq x_0 v(t)$$