Math 211-003: Assignment 6

Due 10/23/2008

These problems are due with work shown by the beginning of class.

#1) Suppose f and g are both of exponential order:

- Show that $\alpha f + \beta g$ is of exponential order, where α and β are constants.

- Show that fg is of exponential order.

#2) Given that $\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$, show that

$$\mathcal{L}\{f^{(n)}\}(s) = s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

#3) Find $\mathcal{L}\{\sin \beta t\}(s)$ for constant β.

#4) Find $\mathcal{L}\{\cos \beta t\}(s)$ for constant β.

#5) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 1 & 1 < t \leq 2 \\ 0 & 2 < t \end{cases}$$

Find $\mathcal{L}\{f\}(s)$.

#6) Find $\mathcal{L}\{t^n\}(s)$ for positive integer n. *(by induction probably)*

#7) Using the laplace transform, change the following IVP into an algebraic equation and solve for $Y(s) = \mathcal{L}\{y\}(s)$:

$$y'' - 3y' + 2y = te^t, \text{ for } y(0) = 0, y'(0) = 1$$