Math 212: Answers to Assignment 1

Section 1.1

#12) The plane spanning $\vec{v_1} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$ and $\vec{v_2} = \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$ contains the origin, so we need a vector normal to $\vec{v_1}$ and $\vec{v_2}$. We will use $\vec{v_1} \times \vec{v_2}$.

$$\vec{v_1} \times \vec{v_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 1 \\ 0 & 3 & 4 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & 1 \\ 0 & 4 \end{vmatrix} + \vec{k} \begin{vmatrix} 3 & -1 \\ 0 & 3 \end{vmatrix}$$

$$= \vec{i}(-1 * 4 - 1 * 3) - \vec{j}(3 * 4 - 0 * 1) + \vec{k}(3 * 3 - (-1) * 0)$$

$$= \begin{pmatrix} -7 \\ -12 \\ 9 \end{pmatrix} = \vec{n}$$

So our choice of $\vec{n}$ and $\vec{p} = \vec{0}$ will give us an equation for the plane:

$$-7(x - 0) - 12(y - 0) + 9(z - 0) = 0 \text{ or } -7x - 12y + 9z = 0$$

Alternatively, you can define the plane by the **span** of $\vec{v_1}$ and $\vec{v_2}$ as

$$\{s\vec{v_1} + r\vec{v_2} | s, r \in \mathbb{R}\} = \{s \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} + r \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} | s, r \in \mathbb{R}\} = \{ \begin{pmatrix} 3s \\ 3r - s \\ s + 4r \end{pmatrix} | s, r \in \mathbb{R}\}$$

#14) We are given a point for our line $\vec{p} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ and a driection vector $\vec{v} = 2\vec{i} - \vec{k} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, so we will use the **point-direction** equation for a line.

$$\vec{l}(t) = \vec{p} + t\vec{v} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2t \\ 2 \\ 1-t \end{pmatrix}$$

#16) We are given two points for our line, $\vec{p_1} = \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix}$ and $\vec{p_2} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}$, so we will use the **point-point** equation for a line.

$$\vec{l}(t) = \vec{v_1} + t(\vec{p_2} - \vec{v_1}) = \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix} + t(\begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} -5 \\ 0 \\ 4 \end{pmatrix}) = \begin{pmatrix} 11t-5 \\ -3t \\ 4-2t \end{pmatrix}$$

#20) We want to show that $\vec{l}(t) = \begin{pmatrix} 1+2t \\ 3t-1 \\ 2+t \end{pmatrix}$ satisfies $5x - 3y - z - 6 = 0$. So we check:

$$5(1+2t) - 3(3t-1) - (2+t) - 6$$

$$= 5 + 10t - 9t + 3 - 2 - t - 6 = 0$$

Since this equation holds for every value t, we know that every point on the line lies in the given plane.

<u>Section 1.2</u>

#2)

$$\vec{a} \bullet \vec{b} = \begin{pmatrix} 2 \\ 10 \\ -12 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix} = 2 * (-3) + 10 * 0 + (-12) * 4 = -54$$

#8)

$$\|\vec{u}\| = \sqrt{5 * 5 + (-1)(-1) + 2 * 2} = \sqrt{30}$$

$$\|\vec{v}\| = \sqrt{1 * 1 + 1 * 1 + (-1)(-1)} = \sqrt{3}$$

$$\vec{u} \bullet \vec{v} = 5 * 1 + (-1) * 1 + 2 * (-1) = 5 - 1 - 2 = 2$$

#14)

$$\vec{u} \bullet \vec{v} = (-1) * 2 + 1 * 1 + 1 * (-3) = -2 + 1 - 3 = -4$$

$$\vec{v} \bullet \vec{v} = 2 * 2 + 1 * 1 + (-3)(-3) = 4 + 1 + 9 = 14$$

$$\vec{u_v} = \frac{\vec{u} \bullet \vec{v}}{\vec{v} \bullet \vec{v}}\vec{v} = -\frac{2}{7} \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -4/7 \\ -2/7 \\ 6/7 \end{pmatrix}$$

<u>Section 1.3</u>

#2.a)

$$\begin{vmatrix} 2 & -1 & 0 \\ 4 & 3 & 2 \\ 3 & 0 & 1 \end{vmatrix} = 2*3*1+(-1)*2*3+0*4*0-3*3*0-0*2*2-1*4*(-1)$$

$$= 6 - 6 + 0 - 0 - 0 + 4 = 4$$

#2.c)

$$\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix} = 1 \begin{vmatrix} 9 & 16 \\ 16 & 25 \end{vmatrix} - 4 \begin{vmatrix} 4 & 16 \\ 9 & 25 \end{vmatrix} + 9 \begin{vmatrix} 4 & 9 \\ 9 & 16 \end{vmatrix}$$

$$= 1(225 - 256) - 4(100 - 144) + 9(64 - 81)$$

$$= 1(-31) - 4(-44) + 9(-17) =$$

$$-31 + 176 - 153 = -8$$

#10) We want to discuss all vectors normal to the given vectors, $\vec{v} = \begin{pmatrix} -5 \\ 9 \\ -4 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$. So we will find a normal vector, then normalize it.

$$\vec{n} = \vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 9 & -4 \\ 7 & 8 & 9 \end{vmatrix} = \vec{i}\begin{vmatrix} 9 & -4 \\ 8 & 9 \end{vmatrix} - \vec{j}\begin{vmatrix} -5 & -4 \\ 7 & 9 \end{vmatrix} + \vec{k}\begin{vmatrix} -5 & 9 \\ 7 & 8 \end{vmatrix}$$

$$= \begin{pmatrix} 81 + 32 \\ -45 + 28 \\ -40 - 63 \end{pmatrix} = \begin{pmatrix} 113 \\ -17 \\ -103 \end{pmatrix}$$

Our other normal vector would be just $\vec{w} \times \vec{v} = -\vec{n}$. So we just need to make $\vec{n}$ a unit vector.

$$\|\vec{n}\| = \sqrt{113^2 + (-17)^2 + (-103)^2} = \sqrt{23,667}$$

so

$$\frac{\vec{n}}{\|\vec{n}\|} = \frac{1}{\sqrt{23,667}} \begin{pmatrix} 113 \\ -17 \\ -103 \end{pmatrix}$$

so our two vectors are $\pm\frac{1}{\sqrt{23,667}} \begin{pmatrix} 113 \\ -17 \\ -103 \end{pmatrix}$.

#13)

$$\vec{u} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \vec{v} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{u} + \vec{v} = \begin{pmatrix} 1 + 2 \\ -2 + (-1) \\ 1 + 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$$

$$\vec{u} \bullet \vec{v} = 1 * 2 + (-2)(-1) + 1 * 2 = 6$$

$$\|\vec{u}\| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$$

$$\|\vec{v}\| = \sqrt{2^2 + (-1)^2 + 2^2} = 3$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 2 & -1 & 2 \end{vmatrix} = \vec{i}\begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} = \begin{pmatrix} -4 + 1 \\ -(2 - 2) \\ -1 + 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$$

#24) To define a plane, we only need to find a point on the plane $\vec{p}$ and the normal vector $\vec{n}$. The question gives us $\vec{p} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, so to get our $\vec{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, we need to recognize that the plane needs to be orthogonal to the line $\vec{l}(t) = \vec{p} + t\vec{v} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$. We can use $\vec{v}$ in the equation of $\vec{l}(t)$ for our $\vec{n}$ since it is the direction of the line. So plugging this information into our equation of the plane yields:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$3(x - 2) - 2(y - (-1)) + 4(z - 3) = 0$$

$$3(x - 2) - 2(y + 1) + 4(z - 3) = 0$$

or if we want $ax + by + cz + d = 0$ form:

$$3x - 6 - 2y - 2 + 4z - 12 = 0$$

$$3x - 2y + 4z - 20 = 0$$