# Math 212: Answers to Assignment 4

## Section 4.3

**#2)** This is just a field of rightward arrows, all of norm 4.

**#4)** The vector field has all arrows pointing away from the x-axis and toward the y-axis. The vertical magnitude grows further from the x-axis, and the horizontal magnitude grows further from the y-axis.

**#14)**

$$(F \circ c)(t) = F(e^{2t}, \log|t|, \frac{1}{t}) = \begin{pmatrix} 2e^{2t} & \frac{1}{t} & -\frac{1}{t^2} \end{pmatrix}$$

$$\vec{c'}(t) = \begin{pmatrix} 2e^{2t} & \frac{1}{t} & -\frac{1}{t^2} \end{pmatrix}$$

$$\text{So } (F \circ c)(t) = \vec{c'}(t) \quad \square$$

**#16)**

$$(F \circ c)(t) = F(\frac{1}{t^3}, e^t, \frac{1}{t}) = \begin{pmatrix} -\frac{3}{t^4} & e^t & -\frac{1}{t^2} \end{pmatrix}$$

$$\vec{c'}(t) = \begin{pmatrix} -\frac{3}{t^4} & e^t & -\frac{1}{t^2} \end{pmatrix}$$

$$\text{So } (F \circ c)(t) = \vec{c'}(t) \quad \square$$

<u>Section 4.4</u>

**#2)**

$$\nabla \bullet V = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xy) = 0 + 0 + 0 = 0 \quad \square$$

**#10)**

$$\nabla \bullet F = \frac{\partial}{\partial x}(y) + \frac{\partial}{\partial y}(-x) = 0 + 0 = 0 \quad \square$$

**#14)**

$$\nabla \times F = \big(\tfrac{\partial}{\partial y}(xy) - \tfrac{\partial}{\partial z}(xz) \quad \tfrac{\partial}{\partial z}(yz) - \tfrac{\partial}{\partial x}(xy) \quad \tfrac{\partial}{\partial x}(xz) - \tfrac{\partial}{\partial y}(yz)\big)$$

$$= \big(x - x \quad y - y \quad z - z\big) = \big(0 \quad 0 \quad 0\big) \quad \square$$

**#18)**

$$\nabla \times F = \big(\tfrac{\partial}{\partial y}(0) - \tfrac{\partial}{\partial z}(-x) \quad \tfrac{\partial}{\partial z}(y) - \tfrac{\partial}{\partial x}(0) \quad \tfrac{\partial}{\partial x}(-x) - \tfrac{\partial}{\partial y}(y)\big)$$

$$= \big(0 - 0 \quad 0 - 0 \quad -1 - 1\big) = \big(0 \quad 0 \quad -2\big) \quad \square$$

## Section 5.1

**#2a)**

$$\int_0^1 \int_{-1}^1 x^4y + y^2 dxdy = \int_0^1 \left[\frac{x^5y}{5} + xy^2\right]_{-1}^1 dy$$

$$= \int_0^1 \left(\frac{1^5y}{5} + 1y^2\right) - \left(\frac{(-1)^5y}{5} + (-1)y^2\right) dy$$

$$= \int_0^1 \frac{2}{5}y + 2y^2 dy = \left[\frac{2}{5}\frac{y^2}{2} + 2\frac{y^3}{3}\right]_0^1$$

$$= \frac{2}{5} \cdot \frac{1}{2} + \frac{2}{3} = \frac{13}{15} \quad \square$$

**#2b)**

$$\int_0^1 \int_0^{\frac{\pi}{2}} y\cos x + 2dxdy = \int_0^1 [y\sin x + 2x]_0^{\frac{\pi}{2}}$$

$$= \int_0^1 (y + 2\frac{\pi}{2}) - (0+0)dy = \int_0^1 y + \pi dy = \left[\frac{y^2}{2} + \pi y\right]_0^1$$

$$= \frac{1^2}{2} + \pi \cdot 1 - 0 - 0 = \pi + \frac{1}{2} \quad \square$$

## Section 5.2

**#2a)**

$$\int_0^1 \int_0^1 x^m y^n dxdy = \int_0^1 \left[\frac{x^{m+1}}{m+1}y^n\right]_0^1 dy$$

$$= \int_0^1 \frac{1}{m+1}y^n dy = \frac{1}{m+1}\left[\frac{y^{n+1}}{n+1}\right]_0^1$$

$$= \frac{1}{(n+1)(m+1)} \quad \square$$