Math 212: Assignment 6

Due 7/18/2008

These problems are due with work shown by the beginning of class.

#1a) Let $\vec{c}\colon [0,1] \to \mathbb{R}^2$ be a line parametrized in **point-point form**. (*i.e.* $\vec{c}(0) = (x_0, y_0)$ *and* $\vec{c}(1) = (x_1, y_1)$). Verify that

$$\int_c x dy - y dx = x_0 y_1 - y_0 x_1$$

#1b) Let D be a triangle with vertices $(x_0, y_0), (x_1, y_1)$, and (x_2, y_2) (in counter-clockwise order). Find the area of D.

#2) Find

$$\iint_D \sin(\sqrt{x^2 + 4y^2}) dA, \text{ where } D = \{x^2 + 4y^2 \leq 4\}$$

#3) Verify that a parallelogram with vertices $(0,0), (b,0), (a,h)$, and $(a+b, h)$ has area bh using **Change of Variables**.
$(a, b, h \in \mathbb{R}, b, h > 0)$

#4) Let $\vec{c}\colon [0,3] \to \mathbb{R}^2$ be

$$\vec{c}(t) = \begin{cases} (t, 0) & t \in [0,1] \\ (1, t-1) & t \in [1,2] \\ (3-t, 3-t) & t \in [2,3] \end{cases}$$

$$\text{Find } \int_c xy dx + xy dy$$

#5) Let $F = \nabla f$ for some $f \in \mathcal{C}^2$ over a simple region $D \in \mathbb{R}^2$. Show that

$$\int_{\partial D^+} F \bullet d\vec{s} = 0$$

#6) Let $D = \{\sin y \leq x \leq \pi + \sin y, \sin x \leq y \leq \pi + \sin x\}$ with density $\rho(x, y) = 1$. Find the **mass** and **center of mass** of D.
(*hint: use* $T(u, v) = (u + \sin v, v + \sin u)$ *w/* $D^* = [0, \pi] \times [0, \pi]$)

#7) Given $D = \{0 \leq x \leq 1, 0 \leq y \leq 1, y - 1 \leq z \leq 1 - y\}$ and $F(x, y, z) = (x^2, y^2, z^2)$, find

$$\iint_{D^+} F \bullet d\vec{S}$$