Math 212: Proof 1

Theorem 1. *If a scalar function f is **even**, then $\nabla f(\vec{0}) = \vec{0}$.*

Proof. We shall begin by defining $g : \mathbb{R}^n \to \mathbb{R}^n$ as follows:

$$g(\vec{x}) = -\vec{x}, \text{ or } g_i(\vec{x}) = -x_i$$

So now we can use the definition of **even** for f to get

$$f(\vec{x}) = f(-\vec{x}) = (f \circ g)(\vec{x})$$

Now we apply the chain rule

$$\nabla f = \nabla f|_g \bullet Dg$$

Since

$$\frac{\partial}{\partial x_j} g_i = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

It follows that $Dg = -I_n$. So

$$\nabla f(\vec{x}) = \nabla f|_{g(\vec{x})} \bullet -I_n = -\nabla f(-\vec{x})$$

So

$$\nabla f(\vec{0}) = -\nabla f(\vec{0}) \Rightarrow \nabla f(\vec{0}) = \vec{0}$$

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