

Math 212: Exam 1

This exam is open book, open notes. Once the test has been opened, you may not consult any person or internet resource (except your instructor and the class website). There is no time limit. The exam is due at 10:00am, Monday June 29th. Show all of your work. This test is pledged, so sign the pledge below upon completion.

Honor Pledge

On my honor, I have neither received nor given any unauthorized aid on this exam.

Signature: _____

#1) A mad scientist has invented a gravity pad. She places a small particle at position $(0,0)$ and spins the gravity pad around the particle, causing the particle to feel acceleration defined by $\vec{a}(t) = (\cos t, \sin t)$.

#a) Assuming that the particle's velocity at $t = 0$ is $(0,0)$, define the motion of the particle as a path function.

#b) Given the assumptions in **(a)**, what is the fastest speed and slowest speed of the particle?

#2) You have 2 dollars to spend on pizza toppings. Pepperoni costs 40 cents per ounce, mushrooms cost 10 cents per ounce, and bell peppers cost 30 cents per ounce. According to wikipedia, the deliciousness of a pizza can be measured by the following function

$$D(x, y, z) = x^2 + yz + z$$

Where x, y, z is the number of ounces of pepperoni, mushrooms, and bell peppers respectively.

#a) How many ounces of each topping should you buy to get the most delicious pizza possible? (you can buy fractions of ounces)

#b) Supposing you're a vegetarian ($x = 0$), how many ounces of the other toppings should you buy to get the most delicious pizza?

#3) Let D be defined as follows:

$$D = \{(x, y) \in \mathbb{R}^2 : x^{\frac{2}{3}} + y^{\frac{2}{3}} \leq 1\}$$

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be

$$f(x, y) = x^{\frac{1}{3}} + y^{\frac{1}{3}}$$

Find the absolute maximum and minimum of f over the region D .

(*Hint*): ∂D can be parametrized by $\vec{c} : [0, 2\pi] \rightarrow \mathbb{R}^2$

$$\vec{c}(t) = (\cos^3 t, \sin^3 t)$$

#4) A C^2 function $f(x, y)$ is called **harmonic** if

$$f_{xx} + f_{yy} = 0$$

Let $x(u, v) = u^2 - v^2$, $y(u, v) = 2uv$, and $f(u, v) = f(x(u, v), y(u, v))$.

Show that $f(u, v)$ is **harmonic** or

$$f_{uu} + f_{vv} = 0$$

#5) Evaluate

$$\iint_D \frac{x^2 + y^2}{xy} dA$$

Over $D = [1, 2] \times [1, 2]$.

#6) Find the relative extrema of $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined as

$$f(x, y) = \sin(x + y) \cos(x - y)$$

(The relationship $\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$ can be helpful)

#7) The temperature in a region of a planet's atmosphere is given by

$$\tau(x, y, z) = x^2 + e^y + \log(z + 1)$$

#a) What direction should one travel from the point $(0, 0, 0)$ to increase the temperature the most rapidly?

#b) Let $\vec{T} = \nabla\tau$. For $t \in [0, 1)$, let

$$\vec{c}(t) = (0, -\log(1 - t), \sqrt{2t - 1} - 1)$$

Show \vec{c} is a flow line for \vec{T} .

#8) Let $\vec{c}: [a, b] \rightarrow \mathbb{R}^3$ for $0 < a < b$ be

$$\vec{c}(t) = (2t, t^2, \log t)$$

Find the arc length of the curve defined by \vec{c} .