MATH 355 HOMEWORK 1

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Problem 1

Part a. We first rearrange the system before writing our augmented matrix:

$$\begin{aligned} 3x + 2y + z &= 1 \\ x + (-y) + z &= 2 \\ 5x + 5y + 2z &= 0. \end{aligned}$$

Now we may read off our augmented matrix:

$$\left(\begin{array}{ccc|c} 3 & 2 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 5 & 5 & 2 & 0 \end{array}\right).$$

Part b. We do the same thing as Part a.

$$\begin{aligned} 0x + 2y + (-2z) &= 3 \\ 3x + (-y) + (-2z) &= -6 \\ x + (-y) + 0z &= -3 \\ x + y + (-2z) &= 0. \end{aligned}$$

becomes

$$\left(\begin{array}{ccc|c} 0 & 2 & -2 & 3 \\ 3 & -1 & -2 & -6 \\ 1 & -1 & 0 & -3 \\ 1 & 1 & -2 & 0 \end{array}\right).$$

Part c. We have to make a choice about ordering the variables here. I'll consider w as coming after z. Then our system becomes

$$\begin{aligned} 2x + (-y) + (-z) + w &= 4 \\ x + y + z + 0w &= -1. \end{aligned}$$

This becomes the augmented matrix

$$\left(\begin{array}{cccc|c} 2 & -1 & -1 & 1 & 4 \\ 1 & 1 & 1 & 0 & -1 \end{array}\right).$$

Part d. Our rearrangement is

$$\begin{aligned} x_1 + (-x_2) + (-x_3) &= 1 \\ (-x_1) + (-2x_2) + x_3 &= 0 \\ x_1 + (-x_2) + (-x_3) &= -1. \end{aligned}$$

This gives the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ -1 & -2 & 1 & 0 \\ 1 & -1 & -1 & -1 \end{array}\right).$$

Problem 2

For these problems note that there are often many different row reductions that will give you the correct answer.

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Part a. We begin with

$$\left(\begin{array}{ccc|c} 3 & 2 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 5 & 5 & 2 & 0 \end{array}\right).$$

First we swap the first and second row. We do this because I like using 1 to clear entries; it will prevent fractions from cropping up as easily. Anyway, we get

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 3 & 2 & 1 & 1 \\ 5 & 5 & 2 & 0 \end{array}\right).$$

Subtract 3 times the first row from the second row:

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 5 & -2 & -5 \\ 5 & 5 & 2 & 0 \end{array}\right).$$

Subtract 5 times the first row from the third row:

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 5 & -2 & -5 \\ 0 & 10 & -3 & 10 \end{array}\right).$$

Subtract 2 times the second row from the third row:

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 5 & -2 & -5 \\ 0 & 0 & -7 & 0 \end{array}\right).$$

And now we're in echelon form and hence can read off the solution. $-7z = 0$, so $z = 0$. Then $5y - 2z = -5$ becomes $5y = -5$, so $y = -1$. Finally, $x - y + z = 2$ becomes $x + 1 = 2$, so $x = 1$. In summary we have a unique solution: $x = 1, y = -1, z = 0$.

Part b. We begin with

$$\left(\begin{array}{ccc|c} 0 & 2 & -2 & 3 \\ 3 & -1 & -2 & -6 \\ 1 & -1 & 0 & -3 \\ 1 & 1 & -2 & 0 \end{array}\right).$$

As in the previous part we interchange the first row with the third row.

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -3 \\ 3 & -1 & -2 & -6 \\ 0 & 2 & -2 & 3 \\ 1 & 1 & -2 & 0 \end{array}\right).$$

Subtract 3 times the first row from the second row:

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -3 \\ 0 & 2 & -2 & 3 \\ 0 & 2 & -2 & 3 \\ 1 & 1 & -2 & 0 \end{array}\right).$$

Now note that the second and third rows are identical. Then we subtract the second row from the third row and move the row of zero entries to the bottom:

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -3 \\ 0 & 2 & -2 & 3 \\ 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

Subtract the first row from the third row:

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -3 \\ 0 & 2 & -2 & 3 \\ 0 & 2 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

Subtract the second row from the third row:

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -3 \\ 0 & 2 & -2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

This augmented matrix is in echelon form, so we can read off the set of solutions. The first row means that $x - y = 3$, so $y = x + 3$. Then the second row gives that $2y - 2z = 3$. In light of the relation $y = x + 3$, we get $2(x + 3) - 2z = 3$, so $2x + 6 - 2z = 3$, so $2x - 2z = -3$, so $z = x + \frac{3}{2}$. I would consider this a complete description of the solution set. If you want to write down a set though you can say $\{(x, x+3, x+\frac{3}{2}) \,|\, x \in \mathbf{R}\}$.

Part c. We begin with

$$\left(\begin{array}{cccc|c} 2 & -1 & -1 & 1 & 4 \\ 1 & 1 & 1 & 0 & -1 \end{array}\right).$$

Again we interchange the rows so that we have a 1 in the upper left:

$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & 0 & -1 \\ 2 & -1 & -1 & 1 & 4 \end{array}\right).$$

Subtract 2 times the first row from the second row:

$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & 0 & -1 \\ 0 & -3 & -3 & 1 & 6 \end{array}\right).$$

This is a matrix in echelon form. Our solutions are given by real numbers x, y, z, w such that $x + y + z = -1$ and $-3y - 3z + w = 6$. If we want to express this relation another way we can say that $z = -1 - x - y$ and $w = 6 + 3z + 3y = 6 - 3 - 3x - 3y + 3y = 3 - 3x$. If we want to write a set we can write $\{(x, y, 1 - x - y, 3 - 3x) | x, y \in \mathbf{R}\}$.

Part d. This one is nice. We begin with

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ -1 & -2 & 1 & 0 \\ 1 & -1 & -1 & -1 \end{array}\right).$$

Subtract the first row from the third row:

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ -1 & -2 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{array}\right).$$

Since $0 \neq -2$ (at least not at this point in your mathematical careers), we have that the system has no solutions. Our matrix still isn't in echelon form though. Next we add the first row the second row:

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{array}\right).$$

Now this matrix is in echelon form, and as we observed before the system has no solutions.

Problem 3

Part a. Let's choose our 3 variables to be x, y, and z (in that order, so the x terms are in the first column, etc.) Then we have

$$x + (-y) + 0z = 5$$
$$3x + (-3y) + 2z = -1$$
$$6x + (-6y) + z = 10.$$

We can write this a bit more cleanly as

$$x - y = 5$$
$$3x - 3y + 2z = -1$$
$$6x - 6y + z = 10.$$

Part b. Let's write the system first as

$$0x + y + 0z = 0$$
$$3x + 0y + 0z = 9.$$

Dropping the variables with coefficient zero out of the system we have

$$y = 0$$
$$3x = 9.$$

Problem 4

Part a.

$$4\vec{v} = 4 \cdot (-1, 3, 2) = (-4, 12, 8).$$

Part b.

$$3\vec{v} - \vec{(}w) = 3 \cdot (-1, 3, 2) - (2, 6, -4) = (-3, 9, 6) - (2, 6, -4) = (-5, 3, 10).$$

Part c.

$$2\vec{w} + 3\vec{i} + 2\vec{k} = 2 \cdot (2, 6, -4) - 3 \cdot (1, 0, 0) + 2 \cdot (0, 0, 1) = (4, 12, -8) - (3, 0, 0) + (0, 0, 2) = (1, 12, -6).$$

Part d.

$$\vec{v} - (-2, -3\pi + 7, 0) = (-1, 3, 2) - (-2, -3\pi + 7, 0) = (-1 + 2, 3 - (-3\pi + 7), 2 - 0) = (1, -4 + 3\pi, 2).$$

Part e.

$$2\vec{v} + 4\vec{j} - \vec{i} - 6\vec{k} = 2 \cdot (-1, 3, 2) + 4 \cdot (0, 1, 0) - (1, 0, 0) - 6 \cdot (0, 0, 1) = (-2, 6, 4) + (0, 4, 0) - (1, 0, 0) - (0, 0, 6) = (-3, 2, -2).$$