Description: Riemannian geometry studies smooth manifolds endowed with Riemannian metrics, which provide a notion of length, angle, volume, and curvature. The information gleaned from this extra structure can give important facts about the manifold's topology, and vice versa (for example, in the Gauss-Bonnet theorem for surfaces). Such relationships between geometry and topology primarily come from curvature, which will be a major topic in this course.

This class will introduce you to the fundamental objects in Riemannian geometry and give you ample opportunity to work with them in concrete examples. I also hope that it tells a nice story, culminating with results that are, in some sense, higher-dimensional analogs of more familiar facts about surfaces. The following is a very rough outline for the topics I hope to cover.

1. Smooth manifolds, tangent spaces, tangent bundles, tensors
2. Riemannian metrics, isometries, model spaces and Lie groups
3. Vector bundles, connections, geodesics, parallel translation
4. Riemannian connections and geodesics, the exponential map, the Hopf–Rinow Theorem
5. Curvature
6. Jacobi fields
7. Cartan–Hadamard Theorem, Bonnet’s Theorem, manifolds of constant sectional curvature

To learn this material well, it will be important to work with many examples. Most of those that we will see come from matrix Lie groups (either directly or indirectly), which is a particularly nice class of spaces to think about.

Textbooks: “Riemannian Geometry” by Manfredo do Carmo
“Riemannian Manifolds: An Introduction to Curvature” by John M. Lee

Homework: I will assign homework regularly; these will be posted on OwlSpace. Your lowest homework score will be dropped in the calculation of your final grade.

Exams: There will be one midterm exam and one comprehensive final exam, both take-home. These will be pledged, and the use of books, notes, or online sources is not allowed unless explicitly stated otherwise.

Evaluation: Your final grade will be determined as the maximum of the following:

25% Homework + 30% Midterm exam + 45% Final exam.
25% Homework + 35% Midterm exam + 40% Final exam.
**Expectations:** I expect you to attend every lecture, though this alone will not be sufficient for deep understanding. Mathematics is learned by grappling with the material for yourself through reading and doing many exercises. I encourage you to read supplementary material and solve exercises from sources other than homework sets. In a similar vein, although you should use office hours to clarify any topics that confuse you, they are also intended for further discussion on material we cover in lecture.

**Disability Support:** Any student with a documented disability who is seeking academic adjustments or accommodations should speak with me during the first week of class. Such discussions will remain confidential, but the student will also need to contact Disability Support Services in the Allen Center.