FINITE GENERATION OF CANONICAL RINGS

Let $X \subset \mathbb{P}^N$ be a smooth algebraic variety i.e. a sub-manifold defined by homogeneous polynomials $P_1, ..., P_l \in \mathbb{C}[z_0, ..., z_N]$. $T_X$ denotes the tangent bundle to $X$ and $\omega_X = \Lambda^{\dim X}(T_X^*)$ is the canonical bundle of $X$. $H^0(\omega_X^m)$ denotes the space of global sections of the $m$-th tensor power of the canonical line bundle so that an element $s \in H^0(\omega_X^m)$ can be written in local coordinates as $f(x_1, ..., x_n)(dx_1 \wedge ... \wedge dx_n)^{\otimes m}$ for some holomorphic function $f$.

The vector spaces $H^0(\omega_X^m)$ play a fundamental role in understanding the geometry of $X$. If $\dim X = 1$, it is well known that $\dim H^0(\omega_X) = g$ is the geometric genus of $X$. If $\dim X = 2$, then the geometry of $X$ was well understood (in terms of the groups $H^0(\omega_X^m)$) by the Italian school of Algebraic Geometry around the beginning of the 20th century. In particular, it is known that if $X$ is not covered by curves of genus 0, then $X$ is birational (i.e. isomorphic outside of a measure zero set) to a unique surface $\tilde{X}$ for which $\omega_{\tilde{X}}$ is semipositive (in the sense that $\deg(\omega_{\tilde{X}}|_C) \geq 0$ for any curve $C \subset \tilde{X}$). In this case, it then follows that the canonical ring $\oplus_{m \geq 0} H^0(\omega_{\tilde{X}}^m)$ is finitely generated. In the 1980’s, by celebrated results of Mori and others, these results were extended from $\dim X = 2$ to the case of $\dim X = 3$.

In this talk I will discuss joint work with Birkar, Cascini and McKernan towards understanding the geometry of algebraic varieties of arbitrary dimension. In particular I will discuss the following:

**Theorem 0.1.** Let $X$ be a smooth projective algebraic variety. Then the canonical ring

$$R(X) = \bigoplus_{m \geq 0} H^0(\omega_X^m)$$

is finitely generated.

Note that this Theorem was independently proven by Siu.