

# Test 2: Math 212 Spring 2008

## Practice Exam

This is a practice exam for Math 212 Exam 2, Spring 2008. It is LONGER than the in class and take home exam will be, but should give you a good idea for the standard types of problems and difficulty level that will be on the exam. Problems like these will constitute roughly 80% of the exam. The rest of the exam will consist of types of problems that you have not seen before. Let me know if you see any typos!!

### Problem 1

Evaluate

$$\int_0^1 \int_y^1 e^{-x^2/2} dx dy$$

### Problem 2

Let  $f(x, y) = \frac{1}{3}x^3 + y\sqrt{2} + 3$  and let  $D$  be the triangle with vertices  $(0, 0)$ ,  $(1, 0)$ , and  $(1, 1)$ . Let  $S$  be the surface given by the graph of  $f(x, y)$  over  $D$ .

(a) Find a parametrization for  $S$ .

(b) Compute

$$\iint_S 4x^2 dS$$

Problem 3

For this problem: Let  $S$  be the disk  $\{(x, y, 5) \mid x^2 + y^2 \leq 1\}$  oriented so that the positive side of  $S$  is up. Let  $C$  be the oriented line from  $(2, 2, 2)$  to  $(0, 0, 0)$ . Also let

$$\begin{aligned}f(x, y, z) &= xy^2z^3, \\ \overline{F}(x, y, z) &= -y\hat{i} + x\hat{j}, \\ \overline{G}(x, y, z) &= 3\hat{i} + \hat{j} + 2\hat{k}\end{aligned}$$

Now, for each of the following quantities, decide if it is **positive**, **negative**, or **zero**. You do not need to justify your answers, no partial credit will be given.

(a)  $\iint_S \nabla \times \overline{F} \cdot d\overline{S}$

(b)  $\int_C \nabla f \cdot d\overline{s}$

(c)  $\iint_S \overline{F} \cdot d\overline{S}$

(d)  $\iint_S \overline{G} \cdot d\overline{S}$

(e)  $\int_{\partial S} \nabla f \cdot d\overline{s}$

Problem 4

(15 points)

Let  $\bar{F} = (z - y, -z, y)$  and  $S$  be the surface given by the conditions  $x^2 + y^2 + z^2 = 1$  and  $x \geq 1/2$ . Orient  $S$  so that the unit normal vectors point away from the point  $(0, 0, 0)$ .

(a) Stokes' Theorem applies here. State the conclusion of Stokes' Theorem for this situation.

(b) Evaluate

$$\iint_S \text{curl}(\bar{F}) \cdot d\bar{S}$$

(Be sure to orient  $\partial S$  properly!)

Problem 5

Let  $W \subset \mathbb{R}^3$  be the region defined by  $x^2 + y^2 + z^2 \leq 1$  and  $y \leq x$ .

(a) Compute

$$\iiint_W (1 - x^2 - y^2 - z^2) dV$$

(b) Compute

$$\iint_{\partial W} \bar{F} \cdot d\bar{S}$$

where  $\bar{F} = (x^3 - 3x, y^3 + xy, z^3 - xz)$  and  $\partial W$  is oriented with the usual convention that the unit normal vectors point out of the region  $W$ .

Problem 6

Use polar coordinates to evaluate

$$\iint_D \sin(x^2 + y^2) dA$$

where  $D$  is the region in the  $xy$ -plane between the circles, centered at the origin, of radius  $\sqrt{\pi/2}$  and  $\sqrt{\pi}$ .

Problem 7

Let  $C$  be the arc between the points  $(0,0)$  and  $(1,1)$  that follows the parabola  $y = x^2$ . Let  $f(x,y) = 2x$ . Evaluate

$$\int_C f ds$$

Problem 8

Let  $D^*$  be the square in the  $uv$ -plane with vertices  $(0,0)$ ,  $(0,1)$ ,  $(1,0)$ , and  $(1,1)$ . Let  $T : D^* \rightarrow \mathbb{R}^2$  be the transformation given by

$$T(u,v) = (x(u,v), y(u,v)) = (2u + v, -u + v)$$

- (a) Precisely describe the image  $D = T(D^*)$  of  $T$ . (Sketching  $D$  and labeling the sketch carefully would be fine.)

- (b) Evaluate

$$\iint_D (x^2 - xy) dA$$

Problem 9

Let  $D$  be the region in the  $xy$ -plane between  $y = x^2$  and  $y = x$ . Suppose the density of  $D$  is given by the function  $\delta(x, y) = x$ .

(a) Calculate the mass of  $D$ .

(b) Calculate the center of mass  $(\bar{x}, \bar{y})$  of  $D$ .

Problem 10

Let  $W$  be the solid body in the first octant of  $\mathbb{R}^3$  bounded by the surface  $x = 4 - y^2$  and the planes  $y = z$ ,  $x = 0$ , and  $z = 0$ .

(The first octant of  $\mathbb{R}^3$  is  $\{(x, y, z) \mid x \geq 0, y \geq 0, z \geq 0\}$ .)

Compute the volume of  $W$ .

Problem 11

Let  $\bar{F} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the vector field

$$\bar{F}(x, y, z) = \left( \frac{-y}{x^2 + y^2 + 2}, \frac{x}{x^2 + y^2 + 2}, 0 \right).$$

(a) Let  $C$  be the unit circle in the  $z = 0$  plane, oriented in the clockwise direction. Evaluate

$$\int_C \bar{F} \cdot d\bar{s}$$

(b) Using your answer from (a), explain why there cannot be any  $C^1$  function  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  with  $\bar{F} = \nabla f$ .