Name: ____ **Section**: _________________

Instructions: You have **3 hours** to complete this exam. You should work alone, without access to the textbook or class notes. You may not use a calculator. Do not discuss this exam with anyone except your instructor.

This exam consists of 9 questions. Except for the first problem (multiple choice), you must show your work to receive full credit. Be sure to **indicate your final answer clearly** for each question.

Note: If you use a major theorem (such as Green's, Stokes', or Gauss' Divergence theorem), you must indicate it. Points will be deducted for failure to indicate the use of a major theorem. **You must clearly indicate each time you use such a theorem.**

Before turning in the exam, be sure to:

- Staple your exam with this cover sheet on top,
- Pledge your exam,
- Write your name and section number above.

The exam is due by **Wednesday, May 10, 4 p.m.** Good luck!

Pledge:

Problem	**Value**	**Score**
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
Total	90	

1. In the following problem, let S be a unit disk in the plane $z = 5$, centered at $(0, 0, 5)$, and oriented upward. Let C be a straight path from $(2, 2, 2)$ to $(0, 0, 0)$. Also, let

$$f(x, y, z) = xy^2z^3,$$
$$\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j},$$
$$\mathbf{G}(x, y, z) = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

Now, for each of the given quantities, decide if it is **positive**, **negative**, or **zero**. (You do not need to justify your answers. No partial credit will be given.)

(a) $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$.

(b) $\int_C \nabla f \cdot d\mathbf{s}$.

(c) $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

(d) $\iint_S \mathbf{G} \cdot d\mathbf{S}$.

(e) $\int_{\partial S} \nabla f \cdot d\mathbf{s}$.

2. (a) Find all critical points of the function $g(x, y) = 2y^3 - 6y + x^2$ in $\mathbb{R}^2$ and classify them.

(b) Let B be the unit ball in $\mathbb{R}^3$, i.e. the set of points (x, y, z) satisfying $x^2 + y^2 + z^2 \leq 1$. Let $f(x, y, z) = 2x + 4y + 6z$. Find the minimum and maximum values of f restricted to B.

3. (a) Let $f(x, y)$ be a C^1 function whose domain is the unit disk in the xy–plane, such that $f(x, y) \geq 0$ everywhere. Suppose that the level set $f(x, y) = 0$ is exactly the unit circle. Let S be the graph of $f(x, y)$, oriented upward, and let

$$\mathbf{F}(x, y, z) = \text{curl}(x^2 - z, e^z + 2x, \pi).$$

Determine $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

(b) Let $g(x, y)$ be a C^1 function whose domain is the unit disk in the xy–plane, such that $g(x, y) \leq 0$ everywhere. Suppose that the level set $g(x, y) = 0$ is exactly the unit circle. Let T be the graph of $f(x, y)$, oriented downward.

Determine $\iint_T \mathbf{F} \cdot d\mathbf{S}$.

4. Evaluate $\int_0^{\sqrt[7]{3^3}} \int_{\sqrt[3]{x}}^{\sqrt[7]{3}} x(\sqrt{1 + y^7})\, dy\, dx.$ Hint: *Consider the region of integration.*

5. Let W be the region in space under the graph of

$$f(x,y) = (\cos y)\exp(1 - \cos 2x) + xy$$

over the region in the xy-plane bounded by the line $y = 2x$, the x axis, and the line $x = \pi/4$.

   (a) Find the volume of W.

   (b) Let $\mathbf{F} = 5x\mathbf{i} + 5y\mathbf{j} + 5z\mathbf{k}$ be the velocity field of a fluid in space. Calculate the flux of $\mathbf{F}$ through the boundary ∂W of W, where W is the region from (a).

6. Let S be a surface in $\mathbb{R}^3$ given as follows. S is the portion of the cylinder $x^2 + y^2 = 9$ lying above $z = 0$, below the graph of $z = \sqrt{x^2 + (y-3)^2}$, and with $y \leq 0$.

   (a) Set up, **but do not evaluate**, an integral giving the surface area of S.

   (b) Suppose S is made of a thin metal whose mass density at any point is given by the function $f(x,y,z) = \sqrt{1 - \frac{y}{3}}$. Find the total mass of S.

7. Let $g(x,y) = xe^y$.

   (a) Compute the second-order Taylor formula for g around $(3,0)$.

   (b) Approximate $2.9e^{0.1}$.

   (c) Compute the directional derivative of g based at $(3,0)$ in the direction of fastest increase.

8. Let $\mathbf{F}(x,y) = 2xy\mathbf{i} + x^2\mathbf{j}$. Show that the line integral of $\mathbf{F}$ around the triangle T (oriented counterclockwise) with vertices $(0,0), (0,1)$ and $(1,1)$ is zero in the following three ways:

   (a) parameterizing T and evaluating the integral directly,

   (b) showing $\mathbf{F}$ is a gradient vector field and explaining why the integral is zero, and

   (c) using Green's Theorem.

9. Compute the flux of the vector field

$$\mathbf{G}(x,y,z) = (xy^2, yz^2 + y, zx^2 + 1)$$

through the unit sphere, oriented outward.