book problems

§2.5 2(a), 3(c), 4, 8(a,c,e)
§3.1 3, 7, 14
§3.2 2, 4, 12, 13

Non-book problem:

1. One says that a set of integers $a_1, \ldots, a_n$ is **relatively prime in pairs** if for all $i \neq j$, $\gcd(a_i, a_j) = 1$.

   a) Show that if $a_1, \ldots, a_n$ are relatively prime in pairs, then

   $$\gcd(a_1, \ldots, a_n) = 1.$$

   b) Show that the converse does not hold when $n \geq 3$; indeed, find the smallest example of three positive integers which are not simultaneously divisible by any $d > 1$ but for which any two have a nontrivial common divisor.

Extra credit problem (2 points):

1. Let $F_1, F_2, \ldots$ be the Fibbonacci numbers discussed in class and in you book, chapter 14. That is, $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n > 2$. Let

   $$\alpha = \tfrac{1}{2}(1 + \sqrt{5}) \quad \beta = \tfrac{1}{2}(1 - \sqrt{5})$$

   Using induction show

   $$F_n = \frac{\alpha^n - \beta^n}{\sqrt{5}}$$