book problems

§3.2 #13(c) If $\gcd(m, n) = 1$ then $gcd(R_m, R_n) = 1$.

Let $x, y \in \mathbb{Z}$ be such that $1 = mx + ny$. Then,

$$\begin{aligned} 1 &= \frac{10 - 1}{9} \\ &= \frac{10^{mx}10^{ny} - 1}{9} \\ &= \frac{10^{mx}10^{ny} - 10^{mx} + 10^{mx} - 1}{9} \\ &= 10^{mx}R_{yn} + R_{mx} \end{aligned}$$

By 13(a), $R_y \mid R_{yn}$ and $R_x \mid R_{mx}$. Therefore, there exist $u, v \in \mathbb{Z}$ such that

$$1 = 10^{mx}uR_n + vR_m$$

and in particular, by Theorem 2.4 in the book, $\gcd(R_m, R_n) = 1$.

Non-book problems:

Extra credit problem: